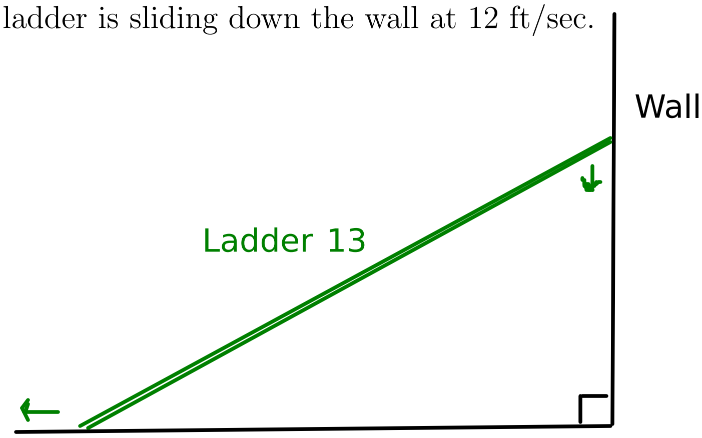
2 Pages **Quiz 7B**, Math 1850 Section 012

10-30-2014 Solutions Name ____________

1. A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving away at the rate of 5 ft/sec. How fast is the top of the ladder sliding down the wall of the house then? (11)

   Draw a picture. The ladder and the house wall and the ground form a right triangle with the ladder as the hypotenuse. Let x be the distance of the base of the ladder to the house. We know that $x' = 5$. Let y be the height where the top of the ladder touches the wall. We want y'.

   Relate x and y: By the Pythagorean Theorem $x^2 + y^2 = 13^2$. Differentiate in time. $2xx' + 2yy' = 0$. We know when $x = 12$, $x' = 5$. What is y? Because $x^2 + y^2 = 13^2$, $12^2 + y^2 = 13^2$ which means that $y = 5$. Therefore $2(12)(5) + 2(5)y' = 0$ so that $y' = -12$. The top of the ladder is sliding down the wall at 12 ft/sec.



(3) 2. Find the linearization $L(x)$ of $f(x) = \sqrt{x^2+9}$, at $a = -4$.

   Recall $L(x)$ is $L(x) = f(a) + f'(a)(x - a)$ which has graph, the tangent line to $y = f(x)$ at $(a, f(a))$. Compute $f(a) = f(-4) = \sqrt{(-4)^2 + 9} = 5$ and $f(x) = (x^2 + 9)^{1/2}$

$$f'(x) = \frac{1}{2}(x^2+9)^{-1/2}2x = \frac{x}{x^2+9}$$

   and $f'(-4) = -4/5$ so that

$$L(x) = 5 - \frac{4}{5}(x - (-4)) = 5 - \frac{4}{5}(x+4)$$

3. Find the differential dy if

(3) (a) $y = \dfrac{2x}{1+x^2}$.

   Differentiate: Apply the quotient rule

$$\frac{dy}{dx} = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2} = \frac{2-2x^2}{(1+x^2)^2}$$

The differential is therefore

$$dy = \frac{2 - 2x^2}{(1 + x^2)^2}\, dx$$

(3) (b) $y = 2\cot\left(\dfrac{1}{\sqrt{x}}\right)$.

Differentiate $y = 2\cot(x^{-1/2})$. We recall that the derivative of $\cot u$ with respect to u is $-(\csc u)^2$. Therefore by the chain rule

$$y' = 2\left(-(\csc(x^{-1/2}))^2(-1/2)x^{-3/2}\right) = \frac{1}{x^{3/2}}(\csc(x^{-1/2}))^2$$

The differential is therefore

$$dy = \frac{1}{x^{3/2}}(\csc(x^{-1/2}))^2\, dx$$