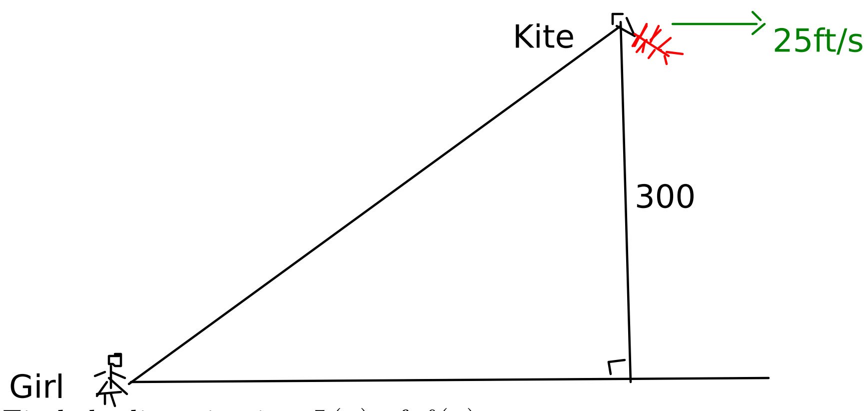
2 Pages	**Quiz 7A**, Math 1850	Section	011
10-30-2014	Solutions	Name	________

1. A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite (9) is 500 ft away from her?

   Draw a picture. The girl, the kite and its string and ground form a right triangle with the string as hypotenuse and the line from the kite straight down to its shadow as one side. We let z be the length of the string that is out because we want to know $z'(t)$. Let x be the distance from the girl to the kite's shadow, because we know that $x'(t) = 25$.

   Relate x and z: By the Pythagorean Theorem $x^2 + 300^2 = z^2$. because the height of the kite is a constant 300 feet. Differentiate in time. $2xx' + 0 = 2zz'$. We know when $x' = 25$ and $z = 500$ at some instant and we want to know z' then. What is x then? From the relation $x^2 + 300^2 = 500^2$ we see that $x = 400$. Substituting into the equation for the derivatives gives $2(400)(25) = 2(500)z'$ so that $z' = 20$. The girl must let out 20 feet of string per second when 500 feet are already out.



(3) 2. Find the linearization $L(x)$ of $f(x) = \tan x$ at $a = \pi$.

   Recall $L(x)$ is $L(x) = f(a) + f'(a)(x - a)$ which has graph, the tangent line to $y = f(x)$ at $(a, f(a))$. Compute $f(a) = f(\pi) = \tan \pi = 0$ and

$$f'(x) = (\sec x)^2$$

   and $f'(\pi) = 1$ so that

$$L(x) = 0 + 1(x - \pi) = x - \pi$$

3. Find the differential dy if

(3) (a) $y = x\sqrt{1 - x^2}$

   Differentiate: Apply the product rule to $y = x(1 - x^2)^{1/2}$.

$$\begin{aligned} \frac{dy}{dx} &= x\frac{1}{2}(1 - x^2)^{-1/2}(-2x) + (1 - x^2)^{1/2} \\ &= \frac{-x^2}{(1 - x^2)^{1/2}} + (1 - x^2)^{1/2} = \frac{-x^2 + 1 - x^2}{(1 - x^2)^{1/2}} = \frac{1 - 2x^2}{(1 - x^2)^{1/2}} \end{aligned}$$

The differential is therefore

$$dy = \frac{1 - 2x^2}{(1 - x^2)^{1/2}}\,dx$$

(3) (b) $y = 4\tan(x^3/3)$

Differentiate: Apply the chain rule. The derivative of $f(u) = \tan u$ is $f'(u) = (\sec u)^2$. Therefore

$$y' = 4(\sec(x^3/3))^2 3x^2/3 = 4x^2(\sec(x^3/3))^2$$

But we were asked for the differential dy and it is

$$dy = 4x^2(\sec(x^3/3))^2\,dx$$