2 Pages **Quiz 6B**, Math 1850 Section 012
10-23-2014 Solutions Name ____________

(12) 1. Find the derivative of y with respect to the appropriate variable.

(a) $y = \ln\left(\dfrac{\sqrt{\theta}}{1+\sqrt{\theta}}\right)$

Simplify $y = \ln\sqrt{\theta} - \ln(1+\sqrt{\theta}) = \frac{1}{2}\ln\theta - \ln(1+\theta^{1/2})$. Now we differentiate: by the chain rule

$$y' = \frac{1}{2}\frac{1}{\theta} - \frac{1}{1+\theta^{1/2}}\frac{1}{2}\theta^{-1/2} = \frac{1}{2\theta} - \frac{1}{2\theta^{1/2}(1+\theta^{1/2})}$$

(b) $y = 5^{\sqrt{s}}$

Use exponential notation. $y = 5^{s^{1/2}}$. Now we differentiate. By the chain rule

$$y' = (\ln 5)5^{s^{1/2}}\frac{1}{2}s^{-1/2} = \frac{(\ln 5)5^{s^{1/2}}}{2s^{1/2}}$$

(because if $f(x) = 5^x$ then $f'(x) = (\ln 5)5^x$).

(c) $y = \sin^{-1}(1-t)$

Differentiate, using the chain rule,

$$y' = \frac{1}{\sqrt{1-(1-t)^2}}(-1) = -\frac{1}{\sqrt{1-(1-t)^2}}$$

(d) $y = \ln(\tan^{-1} x)$

Differentiate, using the chain rule,

$$y' = \frac{1}{\tan^{-1} x}\,\frac{1}{1+x^2} = \frac{1}{(1+x^2)\tan^{-1} x}$$

(4) 2. Use logarithmic differentiation to find the derivative dy/dx if $y = \dfrac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$

Take ln of both sides and simplify

$$\begin{aligned}
\ln y &= \ln\left(\frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}\right)\\
&= \ln\left(x\sqrt{x^2+1}\right) - \ln[(x+1)^{2/3}]\\
&= \ln x + \ln(\sqrt{x^2+1}) - \frac{2}{3}\ln(x+1) = \ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1)
\end{aligned}$$

Differentiate both sides in x. By the chain rule

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\frac{1}{x^2+1}2x - \frac{2}{3}\frac{1}{x+1} = \frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)}$$

so that

$$\frac{dy}{dx} = \left(\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)}\right) y = \left(\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)}\right) \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$$

(4) 3. If $r + s^2 + v^3 = 12$, $dr/dt = 4$ and $ds/dt = -3$, find dv/dt when $r = 3$ and $s = 1$?

Differentiate in t: by the generalized power rule $dr/dt + 2s(ds/dt) + 3v^2(dv/dt) = 0$. We know $dr/dt = 4$ and $ds/dt = -3$ and so we have $4 + 2s(-3) + 3v^2(dv/dt) = 0$ or

$$4 - 6s + 3v^2(dv/dt)$$

We further know that $r = 3$ and $s = 1$ and so from the original equation we know $3 + 1^2 + v^3 = 12$ or $v^3 = 8$ or $v = 2$. Substituting into the above equation involving dv/dt gives:

$$4 - 6(1) + 3(2)^2(dv/dt) = 0$$

so that $dv/dt = 1/6$.