2 Pages **Quiz 6A**, Math 1850 Section 011
10-23-2014 Solutions Name ____________

(9) 1. Find the derivative of y with respect to the appropriate variable.

(a) $y = \ln\left(\dfrac{e^\theta}{1+e^\theta}\right)$
Simplify $y = \ln e^\theta - \ln(1+e^\theta) = \theta - \ln(1+e^\theta)$ and now differentiate.

$$y' = 1 - \frac{1}{1+e^\theta}e^\theta = 1 - \frac{e^\theta}{1+e^\theta} = \frac{1}{1+e^\theta}$$

(b) $y = 2^{(s^2)}$
By the chain rule

$$y' = (\ln 2)2^{s^2}2s = (2\ln 2)s2^{s^2}$$

(because if $f(x) = 2^x$ then $f'(x) = (\ln 2)2^x$).

(c) $y = \sin^{-1}\sqrt{2}x$
Differentiate, using the chain rule,

$$y' = \frac{1}{\sqrt{1-(\sqrt{2}x)^2}}\sqrt{2} = \frac{\sqrt{2}}{\sqrt{1-(\sqrt{2}x)^2}} = \sqrt{\frac{2}{1-2x^2}}$$

(d) $y = \tan^{-1}(\ln x)$
Differentiate, using the chain rule,

$$y' = \frac{1}{1+(\ln x)^2}\frac{1}{x} = \frac{1}{x(1+(\ln x)^2)}$$

(4) 2. Use logarithmic differentiation to find the derivative dy/dx if $y = \sqrt{\dfrac{(x+1)^{10}}{(2x+1)^5}}$

Take ln of both sides and simplify

$$\begin{aligned}
\ln y &= \ln\left(\sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}\right) \\
&= \frac{1}{2}\ln\left(\frac{(x+1)^{10}}{(2x+1)^5}\right) \\
&= \frac{1}{2}(\ln(x+1)^{10} - \ln(2x+1)^5) = 5\ln(x+1) - \frac{5}{2}\ln(2x+1)
\end{aligned}$$

Differentiate both sides in x

$$\frac{1}{y}\frac{dy}{dx} = \frac{5}{1+x} - \frac{5}{2}\frac{1}{2x+1}2 = \frac{5}{1+x} - \frac{5}{2x+1}$$

so that

$$\frac{dy}{dx} = \left(\frac{5}{1+x} - \frac{5}{2x+1}\right)y = \left(\frac{5}{1+x} - \frac{5}{2x+1}\right)\sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}$$

(4) 3. If $x^2y^3 = 4/27$ and $dy/dt = 1/2$, then what is dx/dt when $x = 2$?

Differentiate in t: by the product rule $x^2dy^3/dt + y^3dx^2/dt = 0$ so that $x^2(3y^2(dy/dt)) + y^3(2xdx/dt) = 0$ or $3x^2y^2(dy/dt) + 2xy^3dx/dt = 0$ We know $dy/dt = 1/2$ and, at least for an instant $x = 2$. From the original equation (with $x = 2$) we see $2^2y^3 = 4/27$ or $y = 1/3$. Substituting into the equation for dx/dt and $dy/dt = 1/2$ we see that

$$3(2^2)(1/3)^2(1/2) + 2(2)(1/3)^3dx/dt = 0 \quad \text{or} \quad \frac{dx}{dt} = \frac{3(2^2)(1/3)^2(1/2)}{2(2)(1/3)^3} = \frac{9}{2}$$

that is $dx/dt = 9/2$ at the instant $x = 2$.