Quiz 5B, Math 1850 Section 012

10-16-2014 Solutions Name ____________________

(5) 1. The position of a body moving on a coordinate line is $s = \frac{25}{t^2} - \frac{5}{t}$ with s in meters and t in seconds.

(a) Find the body's displacement and average velocity for the time interval $1 \leq t \leq 5$.

The displacement for the time interval $1 \leq t \leq 5$ is $s(5) - s(1) = 0 - 20 = -20$ The body is displaced 20 meters in the negative direction. The average velocity is the displacement divided by the time elapsed: $-20/(5-1) = -5$ The average velocity is 5 meters per second in the negative direction.

(b) Find the speed and acceleration of the body at time $t = 5$.

Differentiate $s = 25t^{-2} - 5t^{-1}$: $s' = -50t^{-3} + 5t^{-2}$ so that $s'(5) = -1/5$ is the velocity; the speed is $|s'(5)| = 1/5$. The acceleration is $s''(t) = 150t^{-4} - 10t^{-3}$ and at time $t = 5$ it is $s''(5) = 6/25 - 2/25 = 4/25$.

(3) 2. Find ds/dt if $s = \frac{\sin t}{1 - \cos t}$

By the quotient rule

$$\begin{aligned} \frac{ds}{dt} &= \frac{(1-\cos t)\frac{d}{dt}\sin t - \sin t\frac{d}{dt}(1-\cos t)}{(1-\cos t)^2} \\ &= \frac{(1-\cos t)\cos t - \sin t(\sin t)}{(1-\cos t)^2} = \frac{\cos t - (\cos^2 t + \sin^2 t)}{(1-\cos t)^2} = -\frac{1}{1-\cos t} \end{aligned}$$

(3) 3. Find dy/dx if $y = \left(1 - \frac{x}{7}\right)^{-7}$

We apply the generalized power rule (or chain rule)

$$y' = -7\left(1 - \frac{x}{7}\right)^{-8}(-1/7) = \left(1 - \frac{x}{7}\right)^{-8}$$

(6) 4. Find the derivatives of the functions

(a) $y = xe^{-x} + e^{3x}$

Apply the product rule to the first term and the chain rule to the second term.

$$y' = x\frac{d}{dx}e^{-x} + e^{-x}\frac{dx}{dx} + e^{3x}\frac{d}{dx}3x = xe^{-x}(-1) + e^{-x} + e^{3x}3 = (1-x)e^{-x} + 3e^{3x}$$

(b) $r = \sec(\sqrt{\theta})\tan\left(\frac{1}{\theta}\right)$

The product rule applies. First we write $r = \sec(\theta^{1/2})\tan(\theta^{-1})$

$$\begin{aligned}\frac{dr}{d\theta} &= \sec(\theta^{1/2})\frac{d}{d\theta}\tan(\theta^{-1}) + \tan(\theta^{-1})\frac{d}{d\theta}\sec(\theta^{1/2}) \\ &= \sec(\theta^{1/2})\sec^2(\theta^{-1})[-\theta^{-2}] + \tan(\theta^{-1})\sec(\theta^{1/2})\tan(\theta^{1/2})[\frac{1}{2}\theta^{-1/2}]\end{aligned}$$

(3) 5. Use implicit differentiation to find dy/dx if $2xy + y^2 = x + y$

The equation determines y as a function of x at least locally. We differentiate in x

$$2x\frac{dy}{dx} + 2y\frac{dx}{dx} + 2y\frac{dy}{dx} = 1 + \frac{dy}{dx} \quad \text{or} \quad (2x + 2y - 1)\frac{dy}{dx} = 1 - 2y$$

and so

$$\frac{dy}{dx} = \frac{1 - 2y}{2x + 2y - 1}$$