2 Pages | **Quiz 4B**, Math 1850 | Section 012
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(5) 1. Find the limit of the rational function $\lim_{x\to\infty} \frac{7x^3}{x^3 - 3x^2 + 6x}$

The limit is of the form ∞/∞ which means we should look for a cancellation. Factor out the highest power x^3 on the bottom.

$$\lim_{x\to\infty} \frac{7x^3}{x^3-3x^2+6x} = \lim_{x\to\infty} \frac{x^3}{x^3}\frac{7}{1-3/x+6/x^2} = \lim_{x\to\infty} \frac{7}{1-3/x+6/x^2} = \frac{7}{1-0+0} = 7$$

(4 ea.) 2. Find the limit

(a) $\lim_{x\to -8^+} \frac{2x}{x+8}$

We want the limit as x goes to -8 from the right and so $x+8>0$ but, as $x+8$ goes to 0, $2x$ goes to -16.

$$\lim_{x\to -8^+} \frac{2x}{x+8} = -\infty$$

(b) $\lim_{x\to\infty} \frac{x^{-1}+x^{-4}}{x^{-2}-x^{-3}}$

Again we must factor the highest power of x from the bottom and that is x^{-2} and factor out the same power from the top.

$$\lim_{x\to\infty} \frac{x^{-1}+x^{-4}}{x^{-2}-x^{-3}} = \lim_{x\to\infty} \frac{x^{-2}}{x^{-2}}\frac{x+x^{-2}}{1-x^{-1}} = \lim_{x\to\infty} \frac{x+x^{-2}}{1-x^{-1}} \left(=\frac{\infty}{1}\right) = \infty$$

(This is equivalent to multiplying top and bottom by x^2.

(4) 3. Define $g(3)$ in a way that extends $g(x) = (x^2-9)/(x-3)$ to be continuous at $x = 3$.

For g to be continuous at $x = 3$ we must have $g(3) = \lim_{x\to 3} g(x)$. Compute therefore

$$\lim_{x\to 3} g(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} \left(=\frac{0}{0}\right) = \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3} \frac{x+3}{1} = 6$$

Therefore we should define $g(3) = 6$.

(4) 4. For what values of a is

$$f(x) = \begin{cases} a^2x - 2a, & x \geq 2 \\ 12, & x < 2 \end{cases}$$

continuous at every x?

Already f is continuous at every $x \neq 2$ because a linear function a^2x-2a continuous and so is the constant function 12. To be continuous at $x = 2$ requires that

$\lim_{x\to 2} f(x) = f(2) = 2a^2 - 2a$ (Check that $f(2) = 2a^2 - 2a$.) In turn this requires the limit from the left and right at $x = 2$ both be $2a^2 - 2a$. Compute therefore

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} a^2x - 2a = 2a^2 - 2a$$

and

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 12 = 12$$

Therefore we need these both to be $2a^2 - 2a$: that is $12 = 2a^2 - 2a$ or $0 = a^2 - a - 6 = (a-3)(a+2)$. Therefore f is continuous at all x if and only if $a = 3$ or $a = -2$. (Otherwise f is not continuous at $x = 2$.)