2 Pages **Quiz 4A**, Math 1850 Section 011
9-25-14 Solutions Name ____________________

(4) 1. Find the limit of the rational function $\lim_{x\to\infty} \frac{2x^3+7}{x^3-x^2+x+7}$

Factor out x^3 which is the highest power of x on the bottom:

$$\lim_{x\to\infty} \frac{2x^3+7}{x^3-x^2+x+7} = \lim_{x\to\infty} \frac{x^3}{x^3} \frac{2+7/x^3}{1-1/x+1/x^2+7/x^3}$$
$$= \lim_{x\to\infty} \frac{2+7/x^3}{1-1/x+1/x^2+7/x^3} = \frac{2+0}{1+0} = 2$$

(4 ea.) 2. Find the limit

(a) $\lim_{x\to 7} \frac{4}{(x-7)^2}$

This limit is of the form 1/0 which is $\pm\infty$ or does not exist. However the expression $4/(x-7)^2$ is always positive and it gets very large positive as $x \to 7$ and so

$$\lim_{x\to 7} \frac{4}{(x-7)^2} = \infty$$

(b) $\lim_{x\to\infty} \frac{2\sqrt{x}+x^{-1}}{3x-7}$

Again we factor out the largest power of x on the bottom and that is x.

$$\lim_{x\to\infty} \frac{2\sqrt{x}+x^{-1}}{3x-7} = \lim_{x\to\infty} \frac{x}{x} \frac{2/\sqrt{x}+x^{-2}}{3-(7/x)} = \frac{0+0}{3-0} = 0$$

3. Define $h(2)$ in a way that extends $h(t) = (t^2+3t-10)/(t-2)$ to be continuous at
(4) $t = 2$.

For h to be continuous at $t = 2$ we must have $h(2) = \lim_{t\to 2} h(t)$. Compute therefore

$$\lim_{t\to 2} h(t) = \lim_{t\to 2} \frac{t^2+3t-10}{t-2} \left(= \frac{0}{0}\right) = \lim_{t\to 2} \frac{(t-2)(t+5)}{t-2} = \lim_{t\to 2} \frac{t+5}{1} = 7$$

Therefore we should define $h(2) = 7$.

(4) 4. For what value of a is

$$f(x) = \begin{cases} x^2-1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

continuous at every x?

Already f is continuous at every $x \neq 3$ because a quadratic is continuous and so is the linear function $2ax$. To be continuous at $x = 3$ requires that $\lim_{x\to 3} f(x) =$

$f(3) = 6a$. (Check that $f(3) = 6a$.) In turn this requires the limit from the left and right at $x = 3$ both be $6a$. Compute therefore

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x^2 - 1 = 9 - 1 = 8$$

and

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2ax = 6a$$

Therefore we need these both to be $6a$: that is $8 = 6a$ or $a = 4/3$. Therefore f is continuous at all x if and only if $a = 4/3$. (Otherwise f is not continuous at $x = 3$.)