2 Pages	**Quiz 2B**, Math 1850	Section	012
9/11/14	Solutions	Name	____________________

(3) 1. If $f(x) = \dfrac{x+3}{x-2}$ then find a formula for $f^{-1}(x)$.

We solve $y = (x+3)/(x-2)$ and solve for x:$y(x-2) = x+3$ and so $yx - x = 2y+3$ or $x(y-1) = 2y+3$ and so $x = (2y+3)/(y-1)$. Interchange x and y: $y = (2x+3)/(x-1)$. Therefore $f^{-1}(x) = (2x+3)/(x-1)$.

(6) 2. Simplify the expression.

(a) $e^{\ln \pi x - \ln 2}$

$$e^{\ln x - \ln y} = \frac{e^{\ln x}}{e^{\ln y}} = \frac{x}{y}$$

(b) $\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$ (Give an exact result; do not use a calculator.)

We need the value of θ, $-\pi/2 \le \theta \le \pi/2$ so that $\sin\theta = 1/\sqrt{2}$. Therefore $\theta = \pi/4$:

$$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{\pi}{4}$$

(3)

3. Solve for y in terms of x.

$$\ln(y-1) - \ln 2 = x + \ln x$$

Exponentiate both sides.

$$\begin{aligned} e^{\ln(y-1)-\ln 2} &= e^{x+\ln x} \\ \frac{e^{\ln(y-1)}}{e^{\ln 2}} &= e^x e^{\ln x} \\ \frac{y-1}{2} &= xe^x \end{aligned}$$

so that $y = 1 + 2xe^x$

(8) 4. For the curve $y = x^2 - 3$, find

(a) the slope at $P(2,1)$.

To find the slope at P we find the slope of the secant lines through $P(2,1)$ and through $Q(2+h, (2+h)^2 - 3)$. The slope is the rise/run:

$$\begin{aligned} \frac{(2+h)^2 - 3 - 1}{2+h-2} &= \frac{4+4h+h^2-4}{h} \\ &= \frac{4h+h^2}{h} = \frac{h(4+h)}{h} = 4+h \end{aligned}$$

As h gets smaller the secant lines approach the tangent line and so the slope must be $\lim_{h\to 0} 4+h = 4$. Therefore the slope of the tangent line is 4.

(b) an equation of the tangent line at $P(2,1)$.

An equation of the tangent line is that of the line through $P(2,1)$ and of slope 4: $y - 1 = 4(x - 2)$ or $y = 4x - 7$.