

2 Pages!  
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**Quiz 2A**, Math 1850 Section 011  
Solutions Name \_\_\_\_\_

- (3) 1. If  $f(x) = 1/x^2$ ,  $x > 0$  then find a formula for  $f^{-1}(x)$ .

We solve  $y = 1/x^2$  and solve for  $x: x^2 = 1/y$  and so  $x = \pm\sqrt{1/y}$  but  $x > 0$  and so  $x = 1/\sqrt{y}$ . interchange  $x$  and  $y$ :  $y = 1/\sqrt{x}$ . Therefore  $f^{-1}(x) = 1/\sqrt{x}$

- (6) 2. Simplify the expression.

(a)  $e^{\ln x - \ln y}$

$$e^{\ln x - \ln y} = \frac{e^{\ln x}}{e^{\ln y}} = \frac{x}{y}$$

(b)  $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$  (Give an exact result; do not use a calculator)

We need the value of  $\theta$ ,  $-\pi/2 \leq \theta \leq \pi/2$  so that  $\sin \theta = -\sqrt{3}/2$ . Therefore  $\theta = -\pi/3$ :

$$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

- (3) 3. Solve for  $y$  in terms of  $x$ .

$$\ln(y-1) - \ln 2 = x + \ln x$$

Exponentiate both sides.

$$\begin{aligned} e^{\ln(y-1) - \ln 2} &= e^{x + \ln x} \\ \frac{e^{\ln(y-1)}}{e^{\ln 2}} &= e^x e^{\ln x} \\ \frac{y-1}{2} &= x e^x \end{aligned}$$

so that  $y = 1 + 2xe^x$ .

- (8) 4. For the curve  $y = x^2 - 2x - 3$ , find

- (a) the slope at  $P(2, -3)$ .

To find the slope at  $P$  we find the slope of the secant lines through  $P(2, -3)$  and through  $Q(2+h, (2+h)^2 - 2(2+h) - 3)$ . The slope is the rise/run:

$$\begin{aligned} \frac{(2+h)^2 - 2(2+h) - 3 - (-3)}{2+h-2} &= \frac{(2+h)^2 - 2(2+h)}{h} \\ &= \frac{4 + 4h + h^2 - 4 - 2h}{h} = \frac{h(2+h)}{h} = 2+h \end{aligned}$$

As  $h$  gets smaller the secant lines approach the tangent line and so the slope must be  $\lim_{h \rightarrow 0} 2+h = 2$ . Therefore the slope of the tangent line is 2.

(b) an equation of the tangent line at  $P(2, -3)$ .

An equation of the tangent line is that of the line through  $P(2, -3)$  and of slope 2:  $y - (-3) = 2(x - 2)$  or  $y = 2x - 7$ .