

Justify your work. A nongraphing calculator is permitted but not needed. The exam is 2 hours and 200 points are possible; the value of each question is indicated in the left margin. There are 9 pages, including this one: check that no pages are missing.

1. Differentiate the function.

(24)

(a)  $f(x) = \frac{2x+1}{x^2-x}$

(b)  $h(x) = 5x^3 \sin x$

(c)  $g(x) = \frac{1}{(x^4+3x)^{2/3}}$

(d)  $f(\theta) = \csc(8\theta)$

2. Evaluate the limit, if it exists.

(15)

(a)  $\lim_{x \rightarrow 4} \frac{3x-12}{x^2-x-12}$

(b)  $\lim_{\theta \rightarrow 0} \frac{\tan(4\theta)}{\theta}$

(c)  $\lim_{x \rightarrow \infty} \frac{-6x+2x^3}{4-x^2-5x^3}$

(9) 3. Find  $dy/dx$  by implicit differentiation.  $x^2 - 5xy + \sqrt{y} = -10$

4. Find all numbers at which the function

(12)

$$f(x) = \begin{cases} 2x+3 & \text{if } x \leq 0 \\ 2-x & \text{if } 0 < x < 2 \\ (x-2)^2 & \text{if } x \geq 2 \end{cases}$$

is discontinuous. Explain your answer using, for example, limits or a sketch of the graph.

(9) 5. Find an equation for the tangent line to the curve  $y = \sqrt{8 + (1/x)}$  at  $(1,3)$ .

6. Use *the definition of the derivative* to find the derivative of the function.

(11)

$$f(x) = \frac{2}{3x-1}.$$

(10) 7. Evaluate the definite integral  $\int_0^{\pi/2} (x\sqrt{x} + 4\cos x) dx$ .

8. Evaluate the indefinite integral.

(18)

(a)  $\int \frac{5t+t^{1/3}}{t^3} dt$

(b)  $\int t^2 \sqrt{5 + t^3} dt$

(12) 9. Find a function  $f(x)$  such that  $f'(x) = 3x + \sec^2 x$  and  $f(1) = 1/2$ .

(8) 10. Find the *derivative* of the function  $g(x) = \int_2^x \frac{t}{t^4 + 7} dt$ .

(14) 11. Find the absolute maximum and minimum of  $f(x) = 4x^2 - x^4$ , on the closed interval  $-2 \leq x \leq 3$ .

(12) 12. Sketch the graph of a function that satisfies all of the given conditions.

$$\begin{aligned} f'(-2) &= 0; f'(0) = 0; \\ f'(x) &< 0 & \text{if } x < -2 \text{ or } x > 0, \\ f'(x) &> 0 & \text{if } -2 < x < 0, \\ f''(x) &> 0 & \text{if } x < -1 \text{ or } x > 1, \\ f''(x) &< 0 & \text{if } -1 < x < 1. \end{aligned}$$

(18) 13. Let  $f(x) = \frac{x-1}{x^2}$

(a) Find the vertical and horizontal asymptotes, if any.

(b) Find the intervals of increase or decrease.

(c) Find the local maximum and minimum values.

(d) Use the above information to sketch the graph  $y = f(x)$ .

(14) 14. Two cars start from the same point. One car leaves at noon and travels west at 60 km/h. The other car leaves at 1 pm and travels south at 50 km/h. At what rate is the distance between the cars increasing at 2 pm?

(14) 15. If 7200 in<sup>2</sup> of material are available to make a box with square base and open top, find the largest possible volume of the box.

(14)