 Test 2, Math 1730-931 Photo ID Check
7/19/13 Solutions to Practice Test Name ______________________

There are five pages, printed two sided. A single formula sheet and *non-graphing* calculator are permitted. There are 109 possible points. One hours is allowed.

1. (16 points) Find the absolute maximum and minimum of $f(x) = x^3 - 3x^2 - 9x + 2$ on [-2,2].

   Find the critical points. Differentiate $f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$. The critical points are where $f'(x) = 0$ or $f'(x)$ is not defined and so $x = 3$ and $x = -1$ are the critical points but only $x = -1$ is in the interval [-2,2]. Evaluate f at the critical points and the end points $f(-2) = 0$, $f(-1) = 7$ and $f(2) = -20$ so that the absolute max is 7 and occurs at $x = -1$ and the absolute minimum value is -20 and it occurs at $x = 2$

2. (8 points) Calculate the second derivative of $f(x) = 2(4-3x)^6$.

   $f'(x) = 12(4-3x)^5(-3) = -36(4-3x)^5$ and $f''(x) = -180(4-3x)^4(-3) = 540(4-3x)^4$ by the extended power rule.

3. Given the function $f(x) = 8x^3 - 2x^4$, find

   (a) (10 points) The critical values (x values only), the x values where the relative extrema occur, the regions in which the function is increasing and those in which f is decreasing.

   Differentiate $f'(x) = 24x^2 - 8x^3 = 8x^2(3-x)$. Critical values occur when $f'(x) = 0$ or $f'(x)$ is not defined so $x = 0$ and $x = 3$ are the critical numbers. If $x < 0$ the $f'(x) > 0$ (because $f'(-1) = 32$ for example) and so f is increasing. If $0 < x < 3$ then $f'(x) > 0$ (because $f'(1) = 16$ for example) and so f is still increasing. If $3 < x$ then $f'(x) < 0$ (because $f'(4) = -128$ for example) and so f is decreasing.

   Fill in the blanks below, writing “none” is there are no values or intervals:

   - x-values of any critical value(s): $x = 0$ and $x = 3$
   - x-values of any relative minimum(s): none
   - x-values of any relative maximum(s): $x = 3$ (by the first derivative test).
   - region(s) where f is increasing $x < 0$ and $0 < x < 3$
   - region(s) where f is decreasing $x > 3$

   (b) (10 points) Find the x-values of any points of inflection, the regions where the function is concave up or down.

   Differentiate $f''(x) = 48x - 24x^2 = 24x(2-x)$ so that inflection points occur when $f''(x) = 0$ and that is at $x = 0$ and $x = 2$. When $x < 0$, $f''(x) < 0$ (because $f''(-1) < 0$, for example)and so the graph is concave down. When $0 < x < 2$ then $f''(x) > 0$ (because $f''(1) > 0$ for example) and so the graph is concave up. When $x > 2$, $f''(x) < 0$ (because $f''(3) < 0$) and so the graph is concave down

   Fill in the blanks below, writing “none” is there are no values or intervals:

- x values of any point(s) of inflection $x = 0$ and $x = 2$
- region(s) where f is concave up $0 < x < 2$
- region(s) where f is concave down $x < 0$ and $x > 2$

4. A pizza shop determines that the hourly profit, in dollars from the sale of x pizza, is $P(x) = 0.2x^2 + 2x$.

   (a) (4 points) Find the profit when the shop sells 30 pizzas hourly.
   $P(30) = 0.2(30)^2 + 2(30) = 240$

   (b) (4 points) Find the marginal profit when the shop sells 30 pizzas hourly.
   $P'(x) = 0.4x + 2$ and so the marginal profit when $x = 30$ is $P'(30) = 0.4(30) + 2 = 14$

   (c) (2 points) Use your answer in Part b) above to determine the following:
   At a sales level of 30 pizzas hourly, the profit would (circle one) **increase decrease** by \$ 14 if one more pizza is sold.

5. Given $f(x) = \dfrac{x^2 + 3x + 2}{3x^2 - 3}$

   (a) (6 points) Find all vertical asymptotes.
   Vertical asymptotes correspond to division by 0 and so when $3x^2 - 3 = 0$ or $3(x-1)(x+1) = 0$ so that $x = 1$ and $x = -1$ are the vertical asymptotes. (Note ± 1 is not correct since these are numbers and not lines.)

   (b) (6 points) Find all horizontal asymptotes (be sure to use limits).
   Horizontal asymptotes occur at $y = 1/3$ because $\lim_{x \to \pm\infty} f(x) = 1/3$

6. (16 points) A high school can sell 300 yearbooks if it charges \$ 70 per book. For every \$ 2 it drops the price by, the school sells 10 more books. The book costs \$ 20 each to produce. Find the price that should be charged to maximize profit.

   The price per book is $70 - 2x$ where x is the number of increments of \$ 2 decrease the school decreases the price. The number of books sold is $300 + 10x$. The revenues is therefore
   $$R(x) = (70 - 2x)(300 + 10x)$$
   The cost is $C(x) = 20(300 + 10x)$ and so the profit is $P(x) = R(x) - C(x) = (70-2x)(300+10x) - 20(300+10x) = (50-2x)(300+10x) = 15000 - 100x - 20x^2$. (Note $-30 \leq x$ (the number of books sold $300 + 10x$ is nonnegative) and $x \leq 35$ (the price $70 - 2x$ is nonnegative).) To maximize P we differentiate $P'(x) = -100 - 40x$ Critical points occur when $P'(x) = 0$ (that is $x = -5/2$) and when $P'(x)$ does not exist but P' exists for all x. At $x = -5/2$ $P''(x) = -40 < 0$ and so P has a relative maximum at $x = -5/2$ and by the max-min Principle 2 this is an absolute max. The price per book is $70 - 2x = 70 - 2(-5/2) = 75$ that is \$ 75 per book maximizes profit.

7. (5 points each) Calculate the following derivatives.

(a) $f(x) = 7e^{x^3+4x}$
$f'(x) = 7e^{x^3+4x}(3x^2 + 4)$ by the chain rule.

(b) $f(x) = (\ln x)e^{5x}$
$f'(x) = (\ln x)e^{5x}5 + (1/x)e^{5x} = e^{5x}(5 \ln x + 1/x)$ by the product rule.

(c) $f(x) = \log_2(6x - 5)$
$f'(x) = \dfrac{1}{\ln 2}\dfrac{1}{6x-5}6 = \dfrac{6}{(\ln 2)(6x-5)}$ because we recall that $\log_2(x) = (1/\ln 2)\ln x$. (This question is not appropriate for the July 2013 test.)

8. (6 points) Find the present values of a bond that will be worth \$ 20,000 in 15 years assuming that interest is 3.4% compounded continuously. (Round your answer to the nearest cent.)

   Here $P(t) = P_0e^{kt}$ where P_0 is the "present value" (unknown here) and $k = 0.034$. We know $P(15) = 20,000$ or $20,000 = P_0e^{(0.034)15} = P_0e^{0.51}$. Solving for P_0 we have

   $$P_0 = 20,000e^{-0.51} \approx 12,009.91$$

   so that the bond is worth approximately \$ 12009.91 now.

9. (6 points) How long will it take the value of a bank account to double, if the interest is 2.8% compounded continuously?

   We solve for t in the equation $2P_0 = P_0e^{0.034t}$. Cancel P_0 and take ln of both sides

   $$\ln 2 = \ln e^{0.034t} = 0.034t$$

   so that $t = \ln 2/0.034 \approx 20.38$. The value of the account will double in approximately 20.38 years.