 Test 2B, Math 1730-931 Photo ID Check
7/20/13 Solutions Name ____________________

There are five pages, printed two sided. A single formula sheet and *non-graphing* calculator are permitted. There are 100 possible points. One hour is allowed.

1. Calculate the following derivatives. (5 ea)

   (a) $f(x) = \dfrac{1}{\sqrt{4+\ln x}} = (4+\ln x)^{-1/2}$

   By the extended power rule (or chain rule) $f'(x) = -\frac{1}{2}(4+\ln x)^{-3/2}\frac{1}{x} = -\dfrac{1}{2x}(4+\ln x)^{-3/2}$

   (b) $f(x) = x^2e^{x+2}$

   By the product rule $f'(x) = x^2e^{x+2} + e^{x+2}2x = e^{x+2}(x^2+2x)$

   (c) $f(x) = \dfrac{e^{3x}}{6+2x}$

   The quotient rule applies here $f'(x) = \dfrac{(6+2x)e^{3x}3 - e^{3x}2}{(6+2x)^2} = \dfrac{e^{3x}[16+6x]}{(6+2x)^2}$

2. Calculate the second derivative of $f(x) = \ln(4+x^2)$. (9)

   The first derivative is $f'(x) = \dfrac{1}{4+x^2}2x = \dfrac{2x}{4+x^2}$ by the chain rule. Therefore, by the quotient rule

$$f''(x) = \frac{(4+x^2)2 - 2x(2x)}{(4+x^2)^2} = \frac{8-2x^2}{(4+x^2)^2}$$

3. Find the absolute maximum and minimum of $f(x) = 3x^2 - \dfrac{1}{3}x^3$ on [0,9]. (Show your work.) (12)

   Here f is a continuous function on a closed and bounded interval and so that the max-min principle 1 applies (page 251). Differentiate $f'(x) = 6x - x^2 = x(6-x)$ so there is a critical point (set $f'(x) = 0$) at $x = 0$ and $x = 6$. Those are the only two critical points because $f'(x)$ is defined everywhere. In addition to the critical points there are the two endpoints $x = 0$ and $x = 9$. Evaluate f at each of the points: $f(0) = 0$; $f(6) = 36$; $f(9) = 0$. Therefore the absolute maximum value of f is 36 taken at $x = 6$ and the absolute minimum value of 0 is taken at $x = 0$ and $x = 9$

4. Given the function $f(x) = x^3 + 3x^2 + 2$, (18)

   (a) Find

   - x-values of any critical value(s):
   - x-values of any relative minimum(s):
   - x-values of any relative maximum(s):

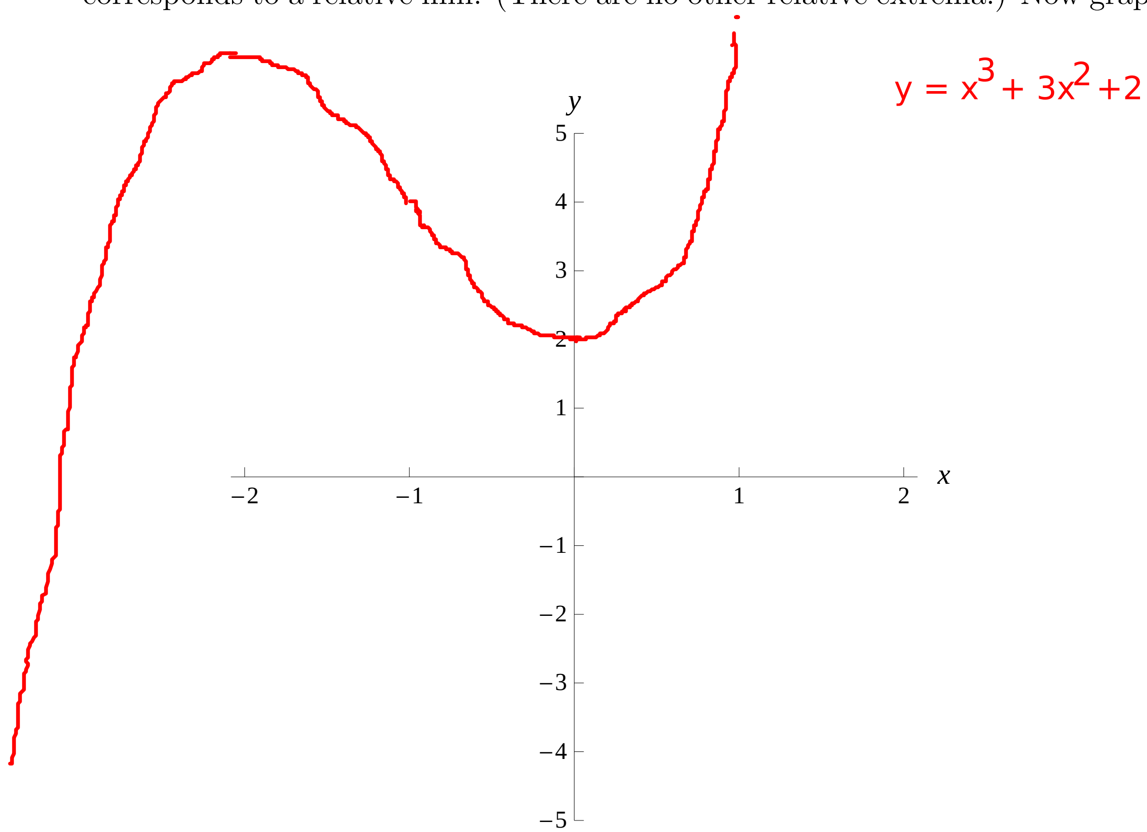
- region(s) where f is increasing
- region(s) where f is decreasing

(b) Sketch the graph of $f(x)$.

Differentiate $f'(x) = 3x^2 + 6x = 3x(x+2)$ so that $x = 0$ and $x = -2$ are critical points (where $f'(x) = 0$) and those are the only critical points because $f'(x)$ is defined everywhere. Check for increasing and decreasing regions.

Interval	Evaluate f'	Increasing or Decreasing
$x < -2$	$f'(-3) = 9 > 0$	Incr
$-2 < x < 0$	$f'(-1) = -3 < 0$	Decr
$0 < x$	$f'(1) = 9 > 0$	Incr

Check that values of f: $f(-3) = -2$; $f(-2) = 6$;$f(-1) = 4$ and $f(0) = 2$. By the first derivative test, we see $x = -2$ corresponds to a relative max and $x = 0$ corresponds to a relative min. (There are no other relative extrema.) Now graph.



(12) 5. If $f(x) = x^4 + 2x^3$ determine

- the x values of any point(s) of inflection
- the region(s) where f is concave up
- the region(s) where f is concave down

Calculate $f'(x) = 4x^3 + 6x^2$; $f''(x) = 12x^2 + 12x = 12x(1 + x)$. The points of inflection correspond to values of x where $f''(x) = 0$ and so $x = 0$ and $x = -1$. Now determine the intervals of cancavity.

Interval	Evaluate f''	Concave Up or Down
$x < -1$	$f''(-2) > 0$	Up
$-1 < x < 0$	$f''(-1/2) < 0$	Down
$1 < x$	$f''(2) > 0$	Up

6. Given $f(x) = \dfrac{x+1}{x^2 + 2x - 3}$ (10)

   (a) Find all vertical asymptotes.

   (b) Find all horizontal asymptotes.

   There are vertical asymptotes where there is a division by 0 and since $x^2 + 2x - 3) = (x+3)(x-1)$ the vertical asympotes are $x = 1$ and $x = -3$. (Note " 1" and "-3" is not the correct answer since asymptotes are lines and should be specified by an equation.) For the horizontal asymptote consider

$$\lim_{x\to\infty} \frac{x-1}{x^2+2x-3} = \lim_{x\to\infty} \frac{x^2}{x^2} \frac{1/x - 1/x^2}{1 + 2/x - 3/x^2} = \lim_{x\to\infty} \frac{1/x - 1/x^2}{1 + 2/x - 3/x^2} = \frac{0}{1} = 0$$

   so that $y = 0$ is the horizontal asymptote at $x = \infty$ and the same calculation shows that $\lim_{x\to-\infty} f(x) = 0$ and so $y = 0$ is the asymptote at $x = -\infty$ as well.

7. A department store sells TV's. Marketing research has determined that if the price of a certain model is $p = 1200 - 4x$ they can sell x units. The department store has costs

$$C(x) = 3000 + 320x$$

   to obtain x units. (13)

   (a) Determine the revenue and the profit.

   The revenue is $R(x) = xp = x(1200 - 4x)$ (price times the number sold) and the profit is $P(x) = R(x) - C(x) = x(1200 - 4x) - 3000 - 320x = -3000 + 880x - 4x^2$.
   (b) Find the price that maximizes the profit.

   Maximize profit: $P'(x) = 880 - 8x$. There is a critical point at $x = 110$ and it is a relative maximum because $P''(110) = -8 < 0$ and so it is an absolute max by max-min Principle 2. The store should sell 110 units and that requires the price $1200 - 4(110) = 760$. The television should be sold for \$ 760 to maximize profit.

8. An initial investment of \$100,000 in a bond fund earns interest compounded continuously at a rate of 2.8% which means that the balance P grows at the rate (11)

$$\frac{dP}{dt} = 0.028P$$

(a) Find the value of the investment after 15 years.
$P(t) = P_0 A e^{kt}$ where $k = 0.028$ and $P_0 = 100,000$ so that $P(t) = 100000e^{0.028t}$. The value after 15 years is $P(15) = 100,000e^{0.028(15)} = 100,000e^{0.42} \approx 152,196.15$ or $ 152,196.15 . ($100,000e^{0.42}$ is also a correct answer.)

(b) How long will it take the value of the account to double?
We want the time t when $P(t) = 200,000$ or $100,000e^{0.028t} = 200,000$. Divide by 100,000 and take the natural log of both sides: $0.028t = \ln 2$ or

$$t = \frac{\ln 2}{0.028} \quad \text{or} \quad t \approx 24.76$$

It takes about 24 and 3/4 years or 24 years and 9 months to double in value.