 Test 1B, Math 1730-931
6/29/13 Solutions Name ___

There are five pages, printed two sided. A single formula sheet and *non-graphing* calculator are permitted. There are 100 possible points

1. Let $f(x) = 3x^2 + 2x + 1$ (12)

   (a) Find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ and simplify.

   Substitute

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)^2 + 2(x+h) + 1 - (3x^2+2x+1)}{h} \\ &= \frac{3(x^2+2xh+h^2) + 2x + 2h + 1 - 3x^2 - 2x - 1)}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 + 2h - 3x^2)}{h} = \frac{6xh + 3h^2 + 2h}{h} = 6x + 3h + 2 \end{aligned}$$

   (b) Use Part (a) above to find $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

   Let h go to 0 in the above expression

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 6x + 3h + 2 = 6x + 2$$

2. Find the following limits or state that the limit does not exist. (12)

   (a) $\lim_{t \to 3} \dfrac{1}{\sqrt{1+t}} = \dfrac{1}{\sqrt{1+3}} = \dfrac{1}{2}$

   (b) $\lim_{x \to -2} \dfrac{x^2 - 4}{x^2 + 3x + 2} = \lim_{x \to -2} \dfrac{(x-2)(x+2)}{(x+2)(x+1)} = \lim_{x \to -2} \dfrac{x-2}{x+1} = \dfrac{-4}{-1} = 4$

   (c) $\lim_{x \to 2} \dfrac{1}{x-2}$

   Here we are getting 1/0 and so the limit will not exist. Can we say more (the limit is $\pm\infty$)?

$$\lim_{x \to 2+} \frac{1}{x-2} = \infty; \lim_{x \to 2-} \frac{1}{x-2} = -\infty$$

   and so the left and right limits do not match and so $\lim_{x \to 2} \frac{1}{x+2}$ does not exist.

3. The value of an investment portfolio grows from an initial value of \$ 100,000 to $V(t) = 100,000 + 1,000t + 11t^2$ dollars where t is time in years. Find (11)

   (a) The value of the account after 10 years.

   $V(10) = 111,100$

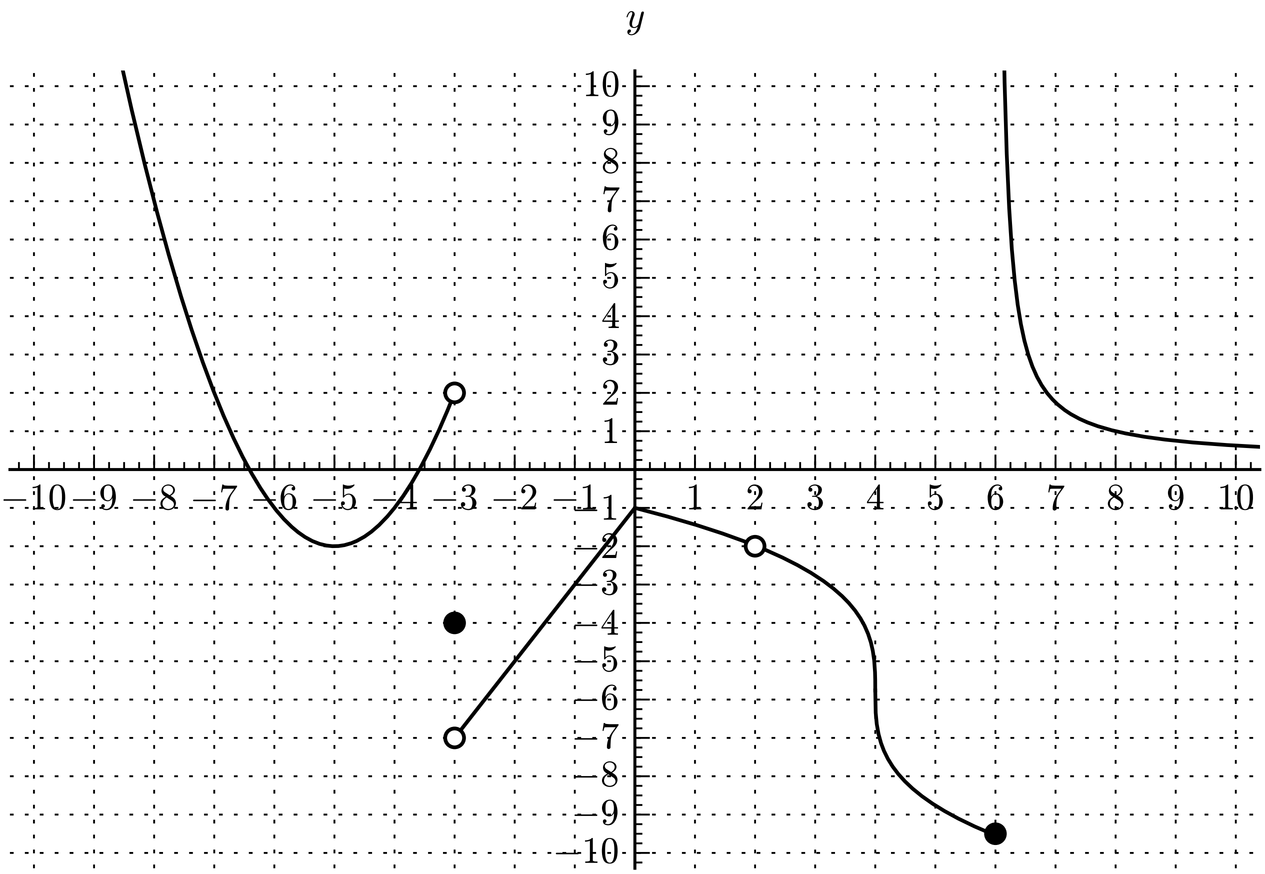
(b) Find the growth rate dV/dt.

$dV/dt = 1,000 + 22t$ is the growth rate.

(c) Find the growth rate after 10 years.

$V'(10) = 1,220$ in dollars per year.

(17) 4. Given the graph of $y = f(x)$ below, answer the related questions. If an expression does not exist then state that fact.



Find, using the graph.

(a) $f(-3) = -4$

(b) $\lim_{x\to -3^-} f(x) = 2$

(c) $\lim_{x\to -3^+} f(x) = -7$

(d) $\lim_{x\to 6^+} f(x) = \infty$

(e) Find all values of x where f is *not* continuous.

f is not continuous at $x = -3$, $x = 2$ (where f is not defined) and $x = 6$,

(f) Is there any value of x where f is continuous but not differentiable?

Yes at $x = 0$

5. Find an equation of the tangent line to the graph of $f(x) = x^2 - 4\sqrt{x}$ at (4,8). (You should use the differentiation techniques like the power rule.) (12)

   Before differentiating it is best to write $f(x)$ in terms of powers of x: $f(x) = x^2 - 4x^{1/2}$. Differentiate $f'(x) = 2x - 2x^{-1/2}$ so that $f'(4) = 8 - (2)4^{-1/2} = 7$. An equation of the tangent line is

$$y - 8 = 7(x - 4) \quad \text{so that } y = 7x - 20$$

6. Differentiate each function. (You should use the differentiation techniques like the power, product and/or quotient rules.) (6 each)

   (a) $f(x) = 3x^9 - 5x^6 + 4x^4 + 12$
   $f'(x) = 27x^8 - 30x^5 + 16x^3$ by the power (and sum) rule.

   (b) $g(x) = 2x^{-3} + 7x^{1/2}$
   $g'(x) = -6x^{-4} + (7/2)x^{-1/2}$ by the power rule.

   (c) $h(x) = \dfrac{6}{x^3} + 8\sqrt{x}$
   Writing h using exponential notation $h(x) = 6x^{-3} + 8x^{1/2}$ so that $h'(x) = -18x^{-4} + 4x^{-1/2}$ by the power rule.

   (d) $f(t) = (t^3 + 13t)(5 + \dfrac{1}{t})$ (Suggestion: Use the product rule)
   First we use exponential notation $f(t) = (t^3 + 13t)(5 + t^{-1})$so that by the product rule $f'(x) = (5 + t^{-1})(3t^2 + 13) + (t^3 + 13t)(-t^{-2}) = (5 + t^{-1})(3t^2 + 13) - (t^3 + 13t)t^{-2}$

   (e) $h(t) = \dfrac{1}{t^4 + 5t + 1}$
   By the quotient rule $h'(t) = \dfrac{-(4t^3 + 5)}{(t^4 + 5t + 1)^2}$

   (f) $g(x) = \dfrac{x^2 + 3x}{7x + 5}$
   By the quotient rule $g'(x) = \dfrac{(2x + 3)(7x + 5) - (x^2 + 3x)7}{(7x + 5)^2}$