 Test 1A, Math 1730-931 Photo ID Check

6/29/13 Solutions Name ______________________

There are five pages, printed two sided. A single formula sheet and *non-graphing* calculator are permitted. There are 100 possible points

1. Let $f(x) = 2x^2 + 3x + 1$ (12)

   (a) Find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ and simplify.

   Substitute

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{2(x+h)^2 + 3(x+h) + 1 - (2x^2 + 3x + 1)}{h} \\ &= \frac{2(x^2 + 2xh + h^2) + 3x + 3h + 1 - 2x^2 - 3x - 1)}{h} \\ &= \frac{2x^2 + 4xh + 2h^2 + 3h - 2x^2)}{h} = \frac{4xh + 2h^2 + 3h}{h} = 4x + 3 + 2h\end{aligned}$$

   (b) Use Part (a) above to find $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

   Let h go to 0 in the above expression

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} 4x + 3 + 2h = 4x + 3$$

2. Find the following limits or state that the limit does not exist. (12)

   (a) $\lim_{t\to 4} \dfrac{1}{\sqrt{5+t}} = \dfrac{1}{\sqrt{5+4}} = \dfrac{1}{3}$

   (b) $\lim_{x\to 2} \dfrac{x^2 - 4}{x^2 - 3x + 2} = \lim_{x\to 2} \dfrac{(x-2)(x+2)}{(x-2)(x-1)} = \lim_{x\to 2} \dfrac{x+2}{x-1} = 4$

   (c) $\lim_{x\to -2} \dfrac{1}{x+2}$

   Here we are getting 1/0 and so the limit will not exist. Can we say more (the limit is $\pm\infty$)?

$$\lim_{x\to -2+} \frac{1}{x+2} = \infty; \lim_{x\to -2-} \frac{1}{x+2} = -\infty$$

   and so the left and right limits do not match and so $\lim_{x\to -2} \frac{1}{x+2}$ does not exist.

3. The value of an investment portfolio grows from an initial value of \$ 100,000 to $V(t) = 100,000 + 2,000t + 22t^2$ dollars where t is time in years. Find (11)

   (a) The value of the account after 10 years.

   The values is $V(10) = 122,200$ dollars.

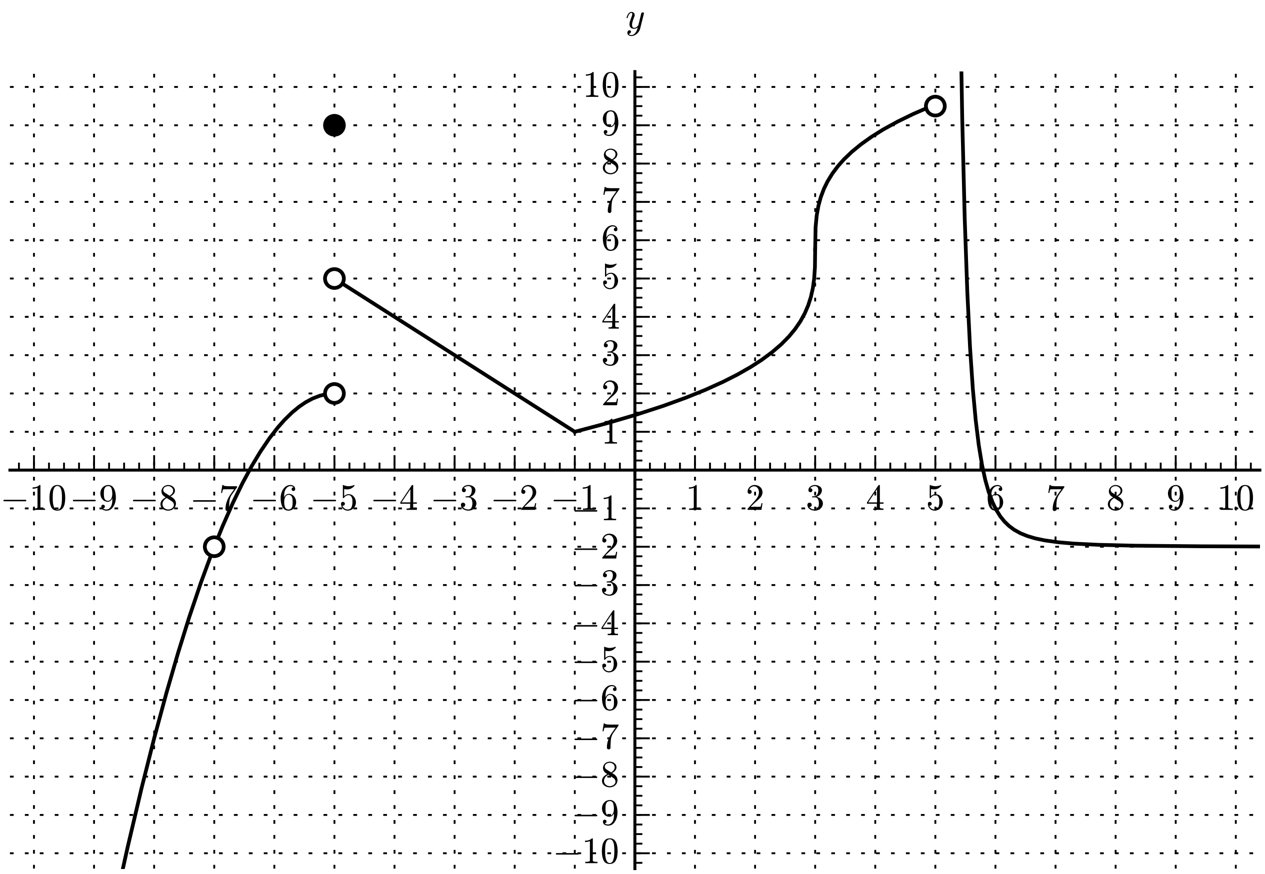
(b) Find the growth rate dV/dt.

The growth rate is $V'(t) = 2,000 + 44t$ by the power and sum rules.

(c) Find the growth rate after 10 years.

The growth rate after 10 years is $V'(10) = 2,440$ which means the account is increasing at 2,440 dollars per year.

(17) 4. Given the graph of $f(x)$ below, answer the related questions. If an expression does not exist then state that fact.



Find, using the graph.

(a) $f(-5) = 9$

(b) $\lim_{x\to -5^-} f(x) = 2$

(c) $\lim_{x\to -5^+} f(x) = 5$

(d) $\lim_{x\to 5^+} f(x) = \infty$

(e) Find all values of x where f is *not* continuous. At $x = -7$, $x = -5$ and $x = 5$.

(f) Is there any value of x where f is continuous but not differentiable? At $x = -1$.

5. Find an equation of the tangent line to the graph of $f(x) = x^2 - \sqrt{x}$ at (4,14). (You should use the differentiation techniques like the power rule.) (12)

   Before differentiating it is best to write $f(x)$ in terms of powers of x: $f(x) = x^2 - x^{1/2}$. Differentiate $f'(x) = 2x - (1/2)x^{-1/2}$ so that $f'(4) = 8 - (1/2)4^{-1/2} = 8 - 1/4 = 31/4$. An equation of the tangent line is

$$y - 14 = \frac{31}{4}(x - 4) \quad \text{so that } y = \frac{31}{4}x - 17$$

6. Differentiate each function. (You should use the differentiation techniques like the power, product and/or quotient rules.) (6 each)

   (a) $f(x) = 3x^8 - 5x^5 + 4x^3 + 12$
   $f'(x) = 24x^7 - 25x^4 + 12x^2$ by the power rule.

   (b) $g(x) = 3x^{-2} + 7x^{1/2}$
   $g'(x) = -6x^{-3} + (7/2)x^{-1/2}$ by the power rule.

   (c) $h(x) = \dfrac{6}{x^4} + 9\sqrt{x}$
   We observe that $h(x) = 6x^{-4} + 9x^{1/2}$ so that $h'(x) = -24x^{-5} + (9/2)x^{-1/2}$ again by the power rule.

   (d) $f(t) = (t^3 + 13)(5t + \dfrac{1}{t})$ (Suggestion: Use the product rule)
   We can use exponential notation: $f(t) = (t^3 + 13)(5t + t^{-1})$ By the product rule $f'(t) = 3t^2(5t + t^{-1}) + (t^3 + 13)(5 - t^{-2})$

   (e) $h(t) = \dfrac{1}{t^4 + 2t + 5}$
   By the quotient rule $h'(t) = \dfrac{-(4t^3 + 2)}{(t^4 + 2t + 5)^2}$.

   (f) $g(x) = \dfrac{x^2 + 4x}{5x + 7}$
   By the quotient rule $g'(x) = \dfrac{(2x + 4)(5x + 7) - 5(x^2 + 4x)}{(5x + 7)^2}$.