Example: Express F(x) = (x^3+5x)^(1/3) as f(g(x)). That is find f and g.

g(x) = x^3+5x
f(x) = x^(1/3)

Example If F(x) = $\frac{4}{x^2+4}$ then express F(x) as f(g(x))

f(x) = 4/x and g(x) = x^2+4

Chain Rule:

$$(f \circ g)' = f'(g(x))g'(x)$$

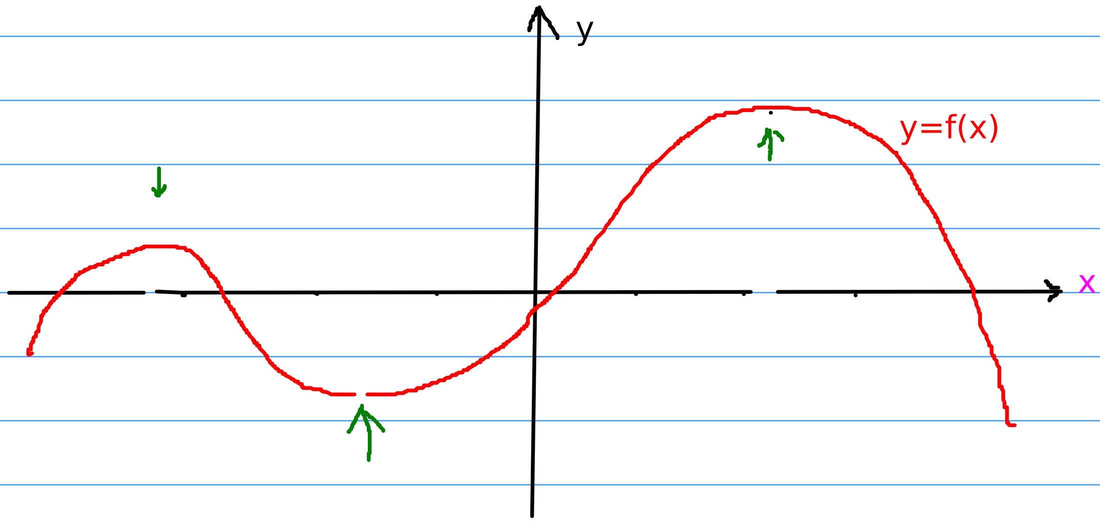
Example Differentiate F(x) = (x^3+5x)^(1/3)

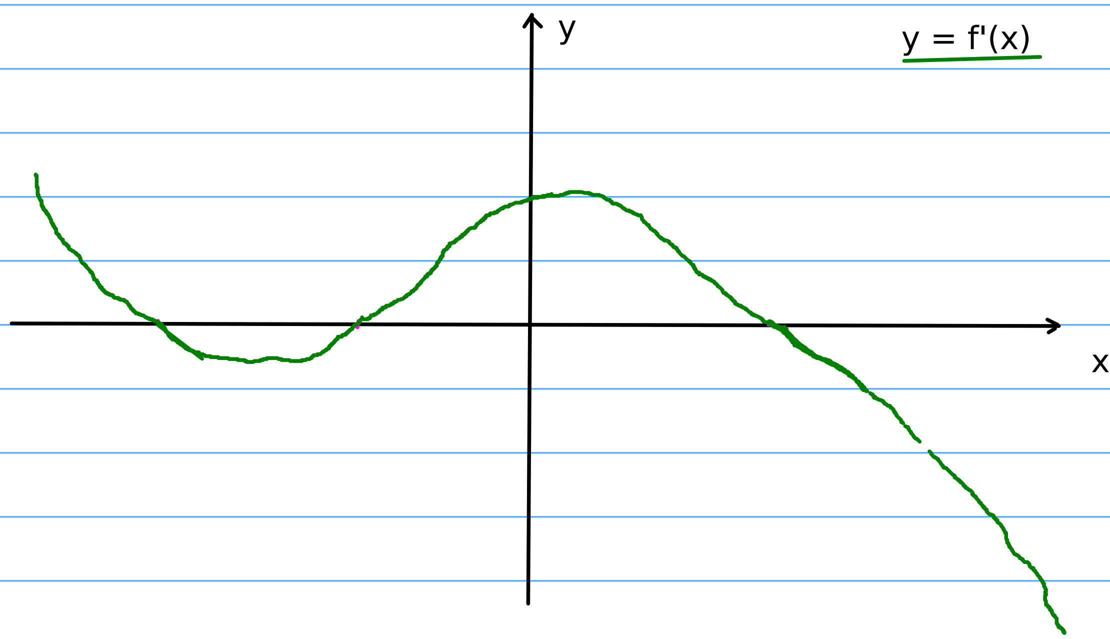
F'(x) = (1/3)(x^3+5x)^(-2/3) ($3x^2$ + 5)

Wednesday, June 26

1.8 Higher Order Derivatives.

The Derivative as a Function



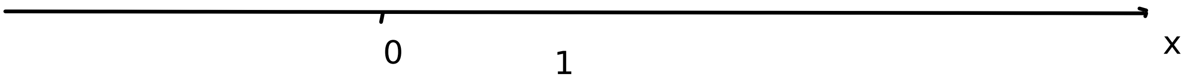


Example: If $f(x) = x^3 + 4x + 8$ then

the first derivative is $f'(x) = \frac{d}{dx} f = 3x^2 + 4$ and

the second derivative is $f''(x) = \frac{d^2}{dx^2} f(x) = 6x$

the third derivative is $f'''(x) = \frac{d^3}{dx^3} f(x) = 6$ $f^{(4)}(x) = 0$



If s(t) is a function of time that indicates where along a straight line an object is then

$$s'(t) = \lim_{h \to 0} \underbrace{\frac{s(t+h) - s(t)}{h}}$$

displacement over the time interval [t,t+h] divided by time elapsed (that is average velocity)

s'(t) is the (instantaneous) velocity.

s''(t) is the acceleration (F = ma Newton's law of motion)

Example: A ball thrown from 6 feet off the ground with initial velocity 64 ft/sec is

$s(t)=-16t^2 + 64 t +6$ feet

above ground level t seconds later.

$v(t) = s'(t) =-32t + 64$

so that the ball continues up for 2 seconds and then falls. When t=2
$v(2)= -32(2)-64=0$

s(2)=70?

$a(t) =s''(t)= -32$ is the acceleration due to gravity. (in feet per second^2)

Summary: First derivative of position (or displacement) is velocity and the second derivative is acceleration.

2.1 Using the first Derivative to Find Maximum and Minimum and Sketch Graphs

decreasing

Definition: A function f is said to be increasing on an interval I if for any a<b , a,b in I

f(a)<f(b)

f(a)>f(b)

If f(x) is differentiable on I and if f'(a)>0 then

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\frac{f(a+h)-f(a)}{h} > 0 \qquad \text{(for } h>0 \text{ or } h<0)$$

so f is increasing.

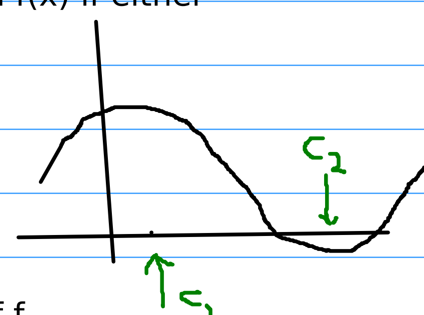
Theorem 1 If f'(x)>0 for all x in I = (c,d) then f is increasing on I

If f'(x)<0 for all x in I = (c,d) then f is decreasing on I

Definition: A number c is a critical number of a function f(x) if either

1) f'(c) = 0

or 2) f' does not exist at c

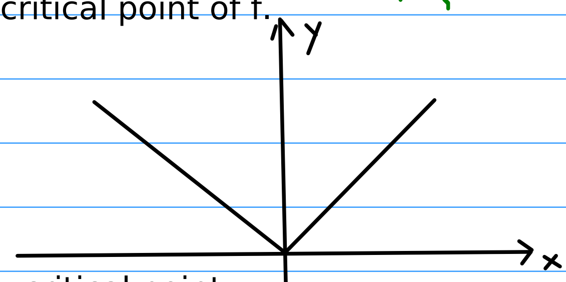


Example: f(x) = x^2-6x + 3 has a critical point at ?

f'(x) = 2x-6

Set to 0 2x-6 = 0 so that x=3 is the only critical point of f.

$$\text{Example } f(x) = |x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$



Critical points of f? f'(0) DNE 0 is the only critical point

The critical points are the points where f can change from being increasing to being decreasing or the reverse (decreasing to increasing)

maximum

Definition: A number c is a relative minimum for f(x) if there exists an open interval I containing c so that f(c) ≤ f(x) for all x in I.

f(c) ≥ f(x)

Fermat's Theorem: If f(x) has a relative maximum or minimum at x=c then c is a critical point (that is: if f'(c) exists then f'(c)=0)

$$f'(c) = \lim_{h \to 0} \frac{f(c+h)-f(c)}{h}$$

In the case c is a relative min, $f(c+h)-f(c) \geq 0$ so that the limit from the right is nonegative and the limit from the left is not positive but the limit from the left and right must be equal.

Example: Find the relative extrema of $f(x) = 2x^3 - 3x^2 - 12x + 6$ and graph the function.

Solution. Differentiate f'(x) = 6x^2 -6x -12 = 6(x^2-x-2) = 6(x-2)(x + 1)

Set 6(x-2)(x+1)=0

The critical points can only occur at x=2 and x = -1 by Fermat's Theorem.

Observe that f'(-2) = 6(4+2-2)>0 so that f is increasing if x<-1

f'(0) =-12 <0 so that f is decreasing -1<x <2

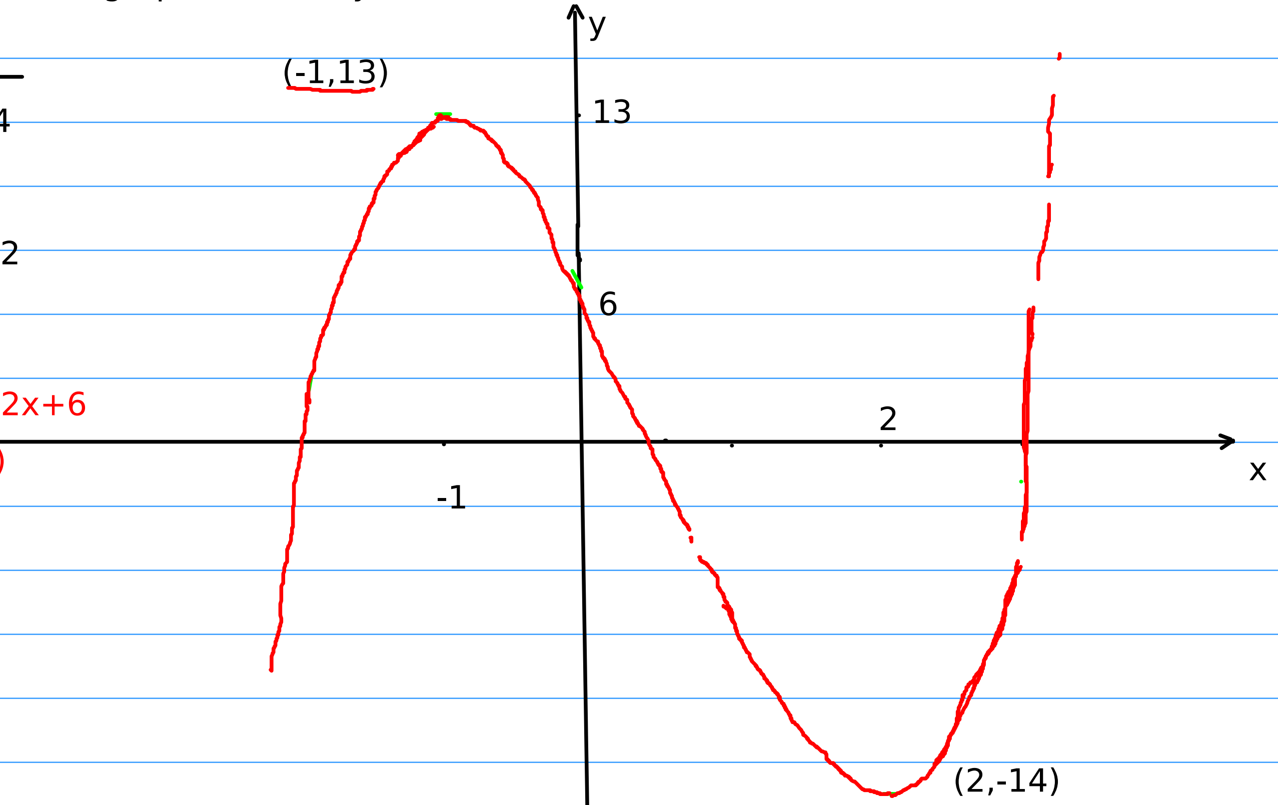
f'(3)=6(4)>0 so that f is increasing. x>2

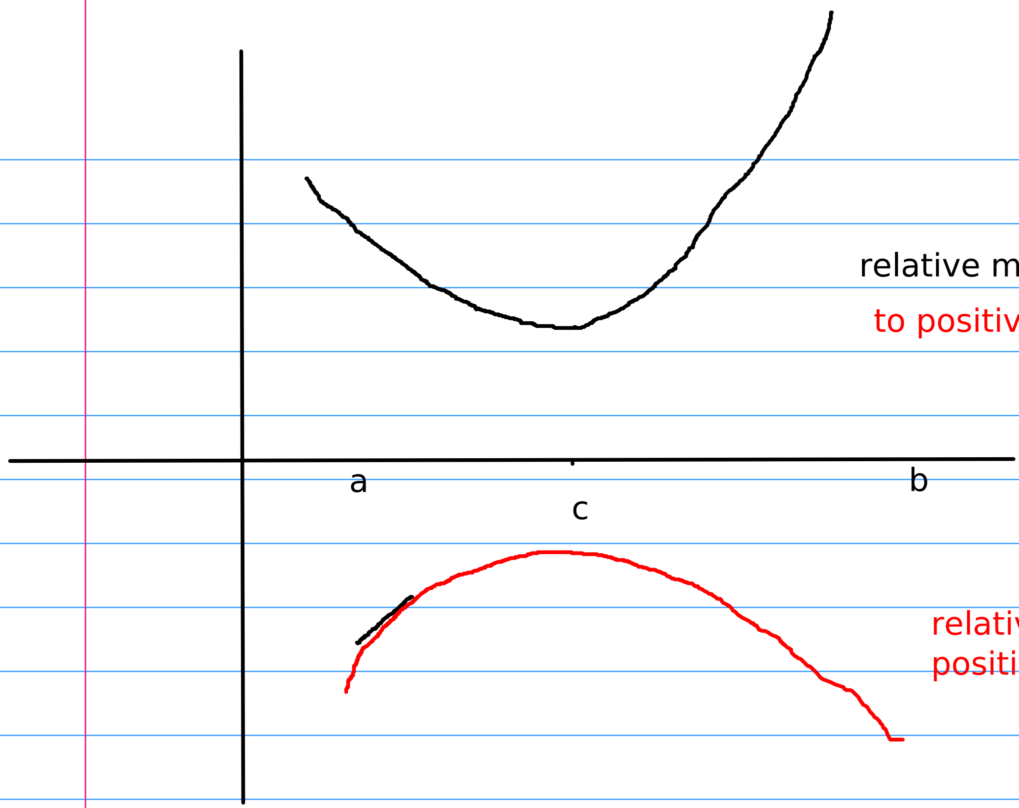
We see that x = -1 is a relative max and x =2 is a relative min and those are the only relative extrema. We make a table of select values of points (x,f(x)) on the graph. Certainly we include the relative extrema

x	y=f(x)	y'
-2	2	24
-1	13	0
0	6	-12
2	-14	0

f(x)=2x^3-3x^2-12x+6

f'(x) = 6(x-2)(x+1)





relative min f'(x) goes from negative
to positive

relative max; f'(x) goes from
positive to negative

Theorem 3 First Derivative Test for Relative Extrema:

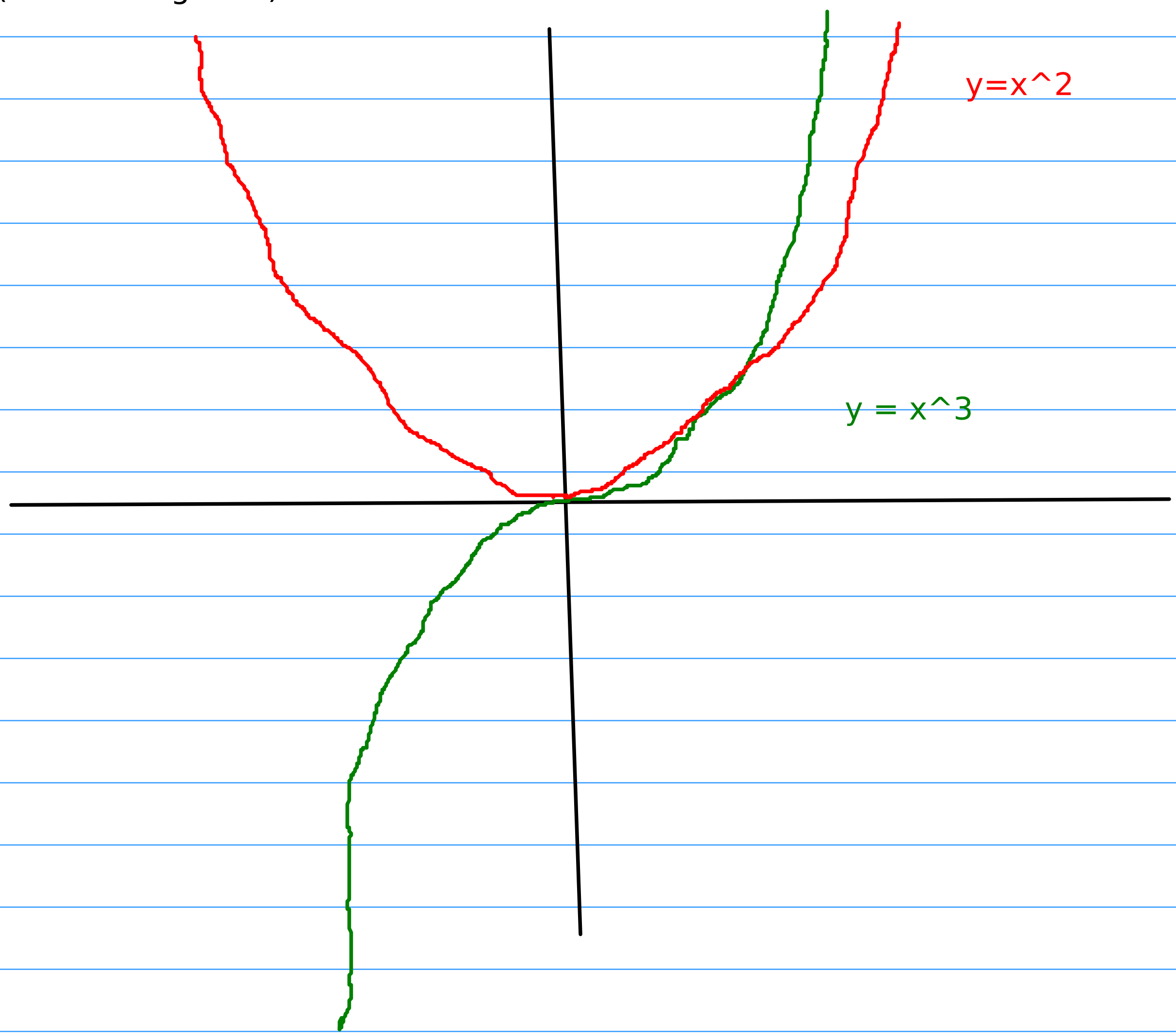
Suppose that c is the only critical point of f(x) in some interval (a,b). a<c<b

1) If f'(x)<0 for x<c and f'(x)>0 for c<x then c is a relative min
2) If f'(x)>0 for x<c and f'(x)<0 for c<x then c is a relative max
3) Otherwise f'(x) does not change sign at c and c is not a relative extrema.

x=0 is a critical point

Example f(x) = x^2 has a critical point when f'(x) = 2x = 0, and that is a relative min because f'(x) =2x goes from negative to positive at x=0.

Example f(x) = x^3 has a critical point when f'(x) = 3x^2 = 0 and that is x=0 but that is not a relative max nor min because f'(x) does not change sign (it is nonnegative) at x = 0.



Example: Find the relative extrema of $f(x) = 4x^3 - 3x^4$

Solution: $f'(x) = 12x^2 - 12x^3 = 12x^2(1-x)$

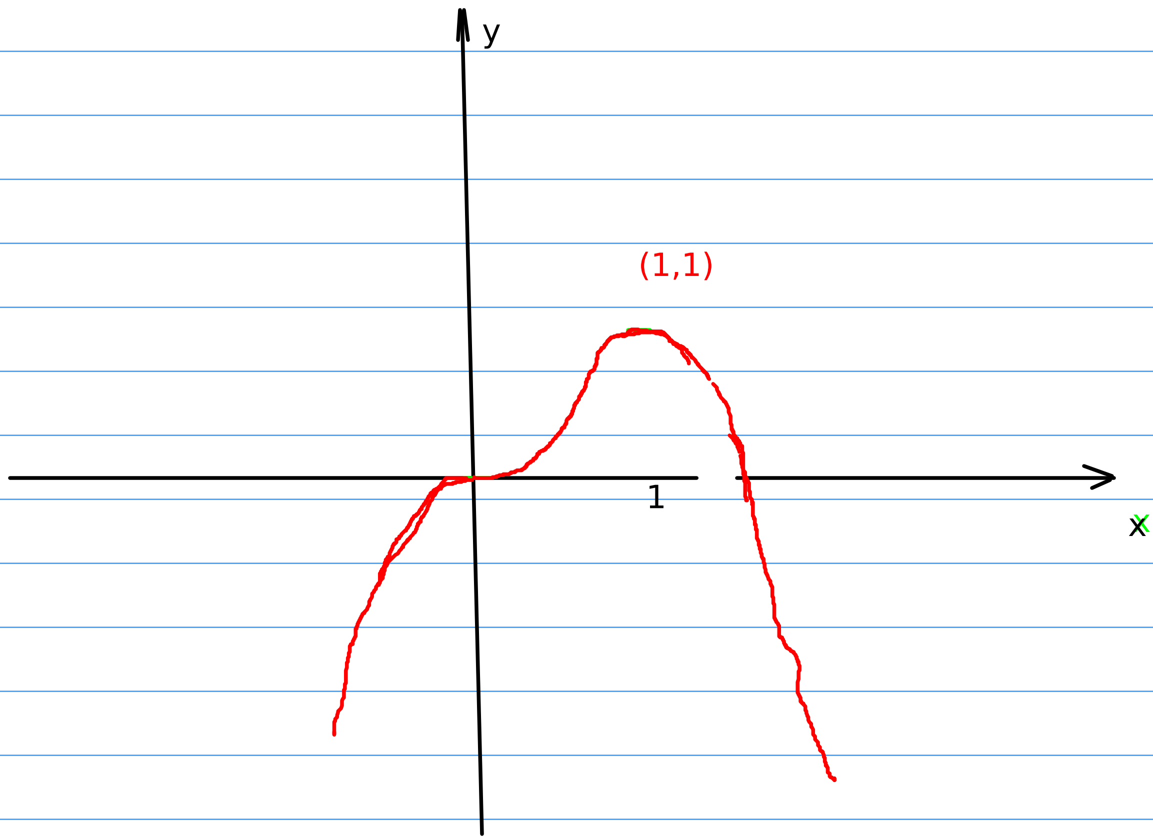
The critical points are where $f'(x) = 0$ and that is at $x = 0$ and $x = 1$.

There are no points where $f'(x)$ does not exist and so there are no other critical points.

Near $x = 0$, $f'(x)>0$ so that $x=0$ is neither a local max nor min.

At $x = 1$ $f'(x)$ goes from being positive to being negative as x increase through 1

($f'(x)>0$ if $x<1$ and $f'(x)<0$ if $x>1$.) Therefore $x = 1$ is a relative max .



2.2 Using Second Derivatives.

If $f''(x) > 0$, $a<x<b$ then f is concave up

If $f''(x)<0$, $a<x<b$ then f is concave down.