Class June 12

Recall from Monday's class

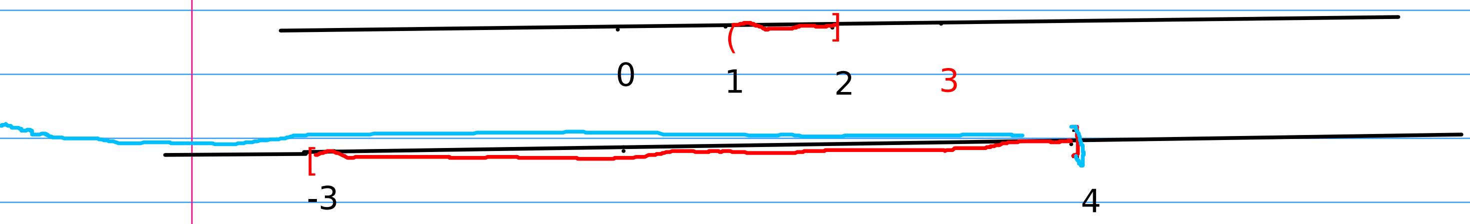
R.3 Domain and Range.

Interval Notation: (1,2] $= \{x: 1<x\leq 2\}$

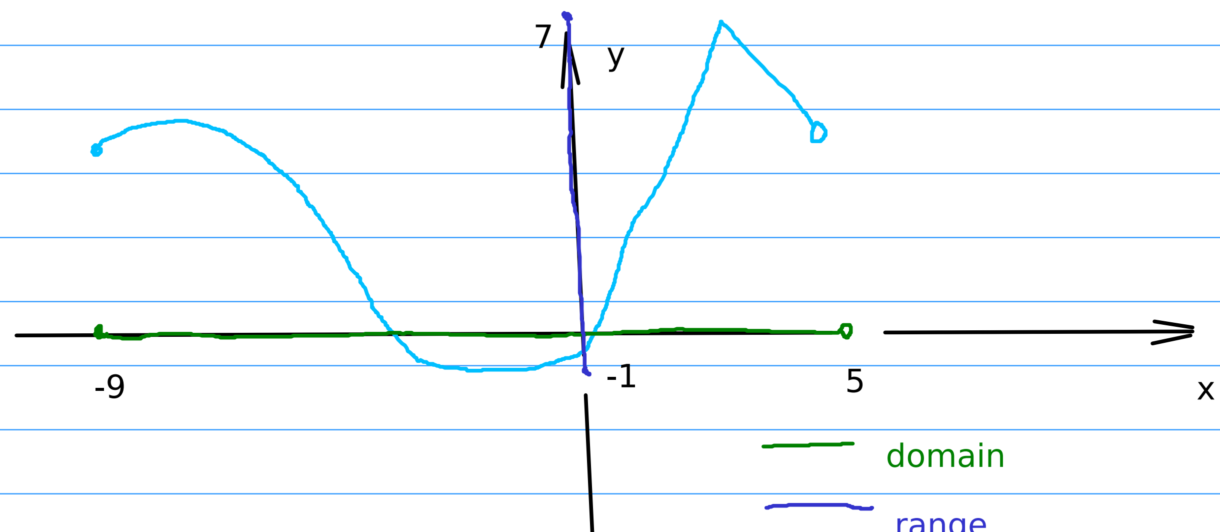
[-3,4] $=\{x| -3\leq x\leq 4\}$

(-3,4)

(- ∞ 4]



Finding the domain and range of a function given its graph:



Domain =[-9,5)

Range =[-1,7]

Of course the domain might be specified, but if not then it is understood that the domain is a large as possible so that the formula for the function makes sense.

Examples: $f(x) = \frac{4-2x}{3-x}$ All x except x = 3

$g(x) = \sqrt{x-5}$

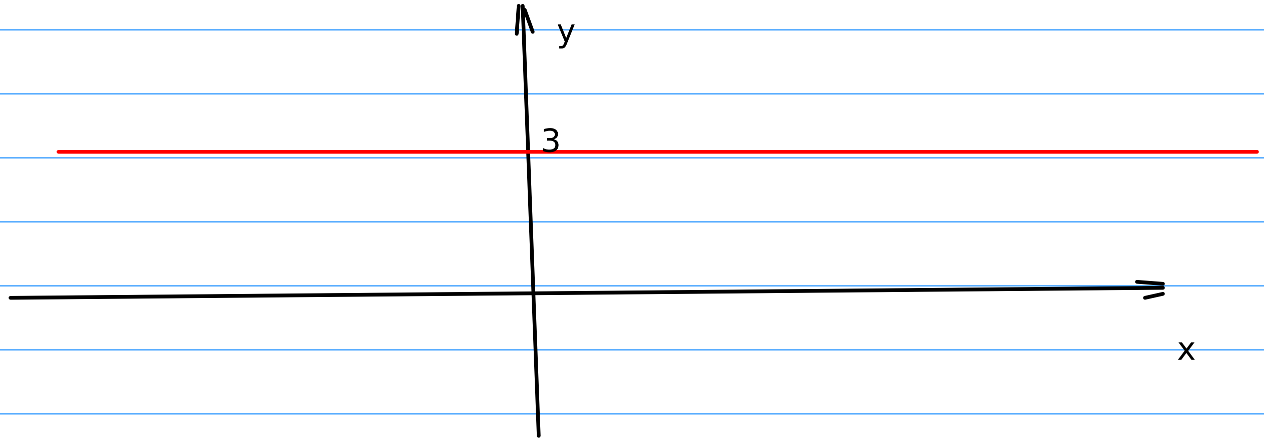
Domain of f(x) $(-\infty, 3) \cup (3, \infty)$

$\frac{3-x}{x-3} = -1$ all x except x =3

Domain of g(x): All $x \geq 5$

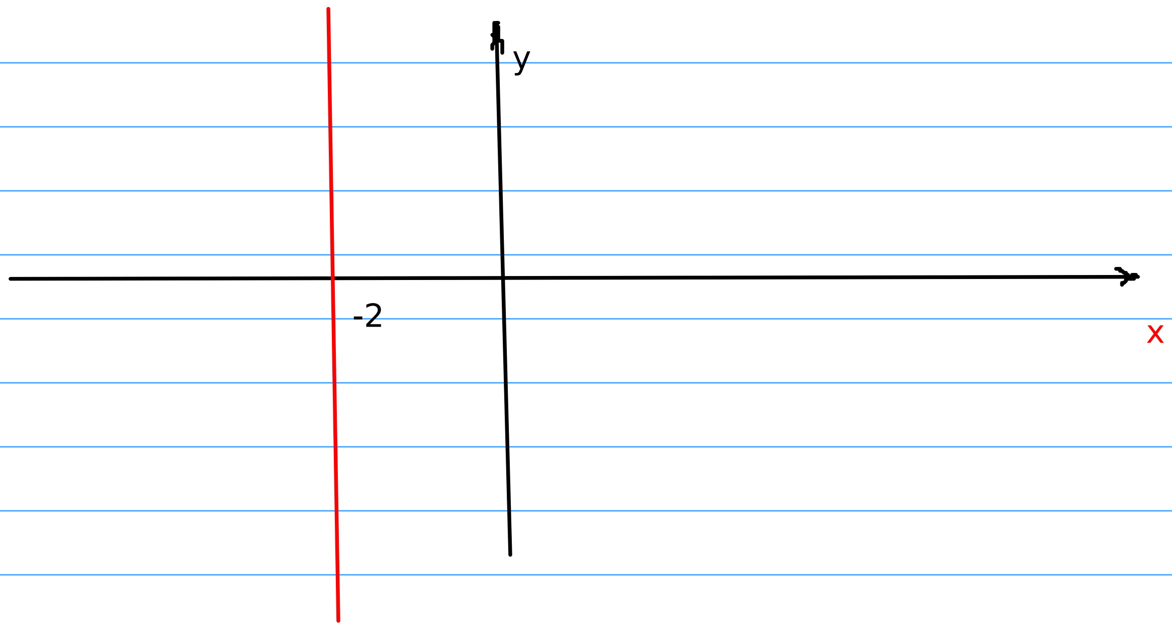
R.4 Slope and Linear Functions

A horizontal line has zero slope. For example the line



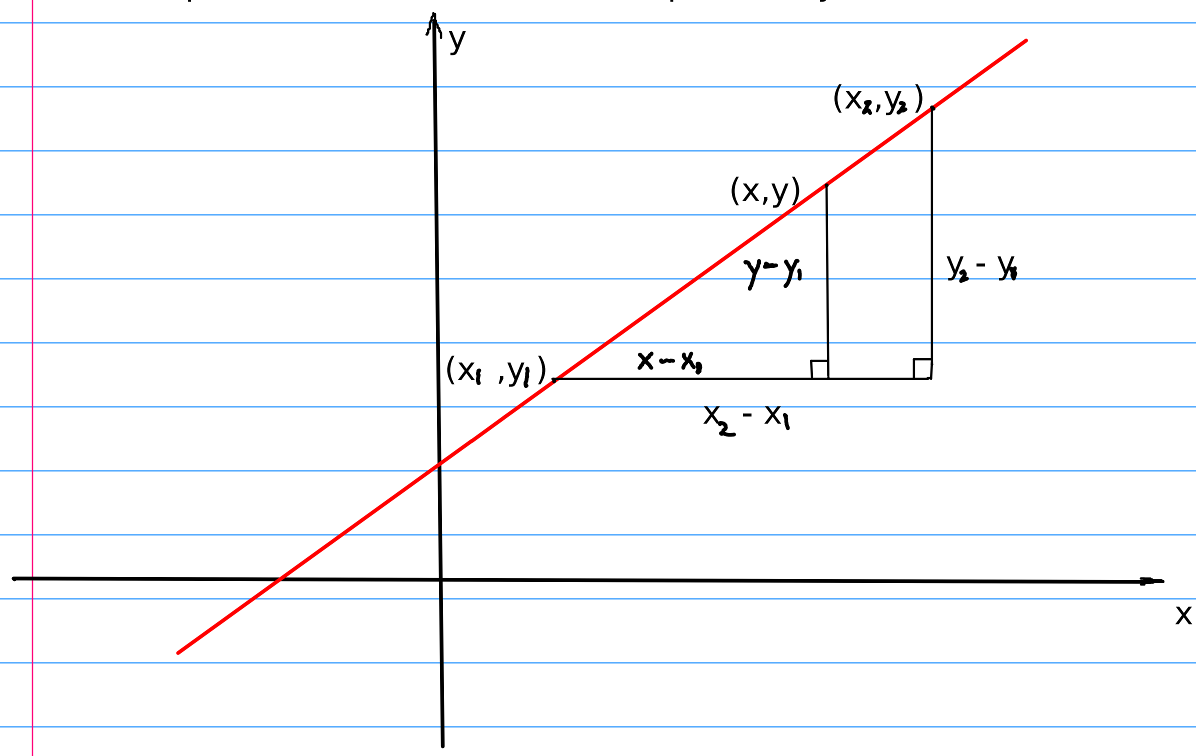
has equation y=3. Note y=f(x) = 3 does not depend on x.

A vertical line does not have a slope or, in other words, slope is not defined for a vertical line. For example



has equation x=-2. This line is a curve which is NOT the graph of a function.

In general a line is determined by two points: If (x_1, y_1) and (x_2, y_2) are two distinct points on a line then all other points (x,y) on the line must satisfy:



The triangles are similar triangles and so the ratio of the side lengths is the same

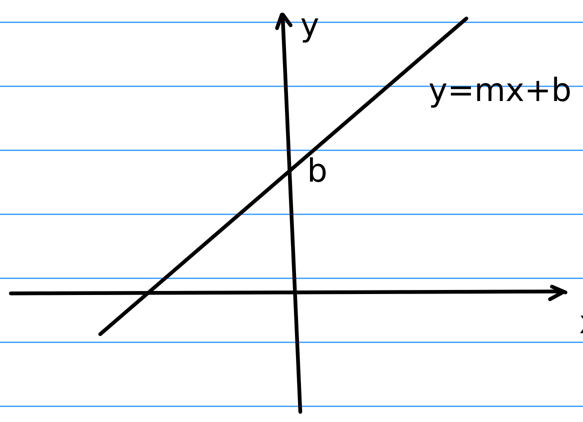
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

We define the slope of the line to be (assuming $x_1 \neq x_2$) m where

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

We get the
Point-Slope Equation of the line : $y - y_1 = m(x - x_1)$

Slope-Intercept Equation of the line $y = mx + b$
(by taking $x_1 = 0$ so that $b = y_1$ is the y-intercept of the line.)



Example Find an equation for the lines
a) through (-1,2) and (1,6).
b) of slope 2/3 and y-intercept - 1
c) of slope 3 and through (2,-4)

a) $\frac{6-2}{1-(-1)} = \frac{4}{2} = 2$

$y - 2 = 2(x - (-1)) = 2(x+1) = 2x + 2$

$y = 2x+4$ (Plug in x=-1 and find y = 2; plug in x=1 and find y = 6)

b) $y = mx + b = (2/3)x + (-1) = (2/3)x - 1$

c) $y - y_1 = m(x - x_1)$

$y - (-4) = 3(x-2)$

$y + 4 = 3(x-2)$

$y = 3x - 10$

The book mentions some interesting examples. The slope for example corresponds to the pitch of a roof and also to the gradient of a road. Linear functions are often the simplest model: for example the population of the USA was about 200 million in 1970 and 300 million in 2010 and so a linear model would suggest that by 2050 the population will be 400 million.

R.5 Nonlinear Functions and Models.

Quadratic models are sometimes more accurate than linear models.

A quadratic is a second order polynomi $y = a x^2 + bx + c$

Observe that if a = 0 then a quadratic is linear (y=bx+c)

y = a x^2 + b x + c

Graphing a quadratic: A quadratic has graph a parabola ($a \neq 0$). which opens up if a>0 and down if a<0. By completing the square

$$y = a x^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

The vertex is at the point $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$ The axis of symmetry is $x = -\frac{b}{2a}$

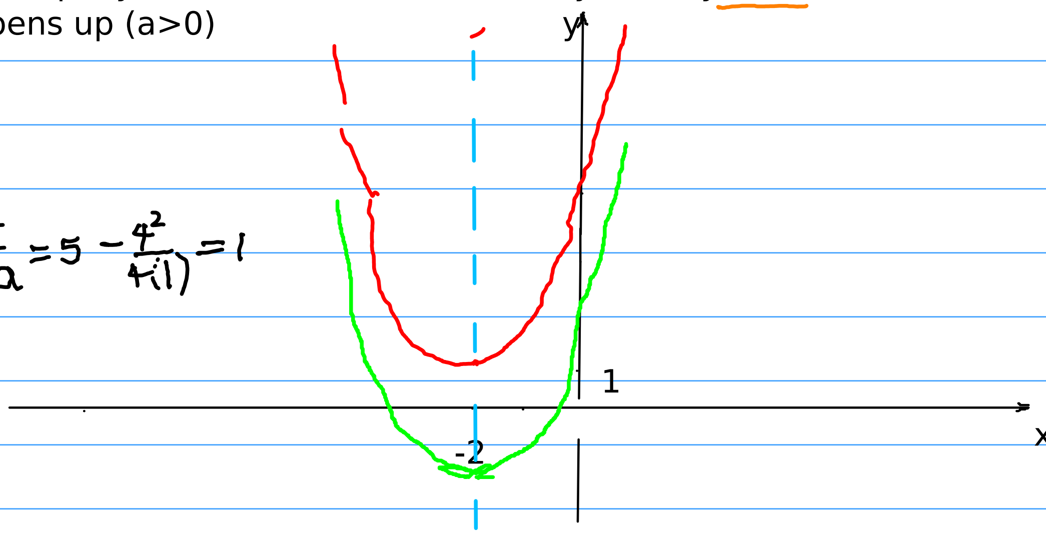
Example $y = x^2 + 4x + 5$ has axis of symmetry $x = -2$ and vertex (-2,1) and it opens up (a>0)

$a = 1$

$b = 4$

$c = 5$

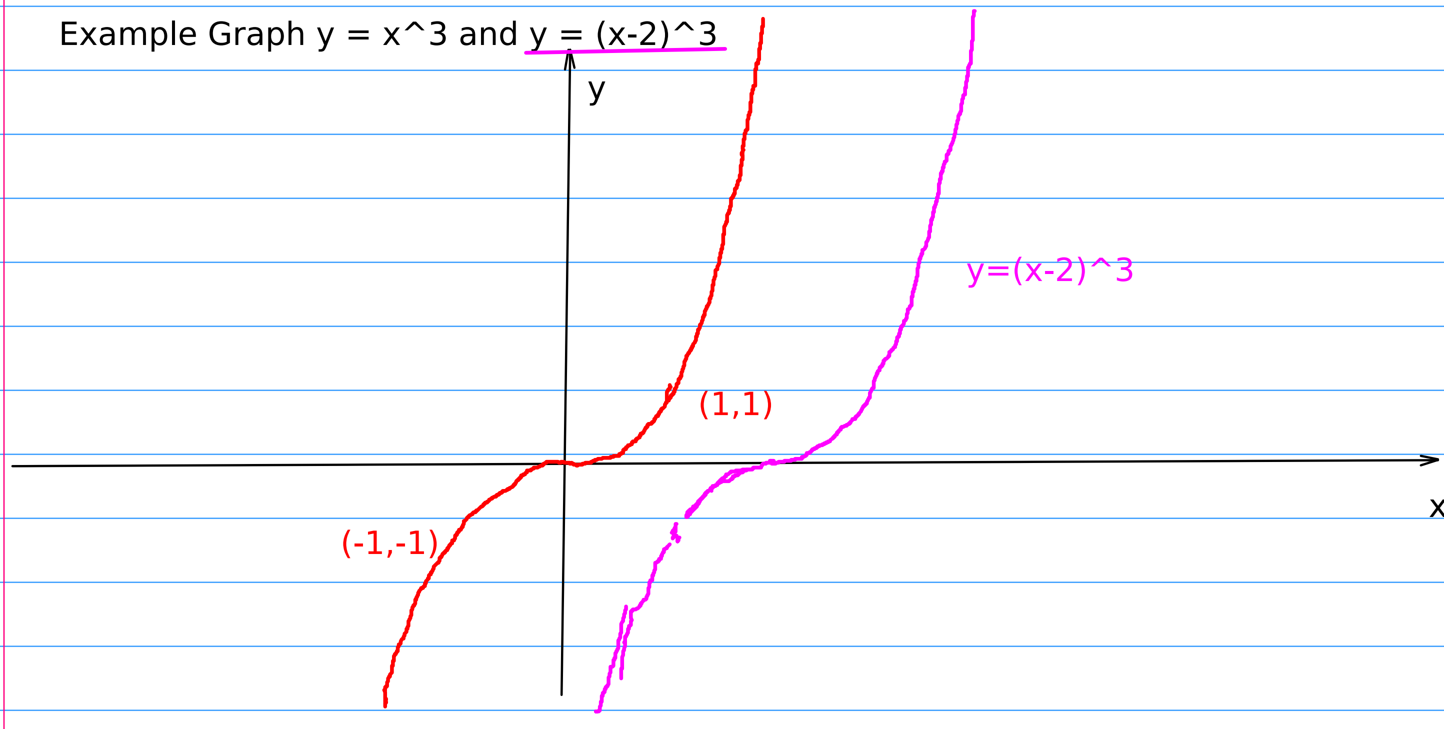
$c - \frac{b^2}{4a} = 5 - \frac{4^2}{4(1)} = 1$



Example What would the graph of $y = x^2 + 4x + 3$ be?

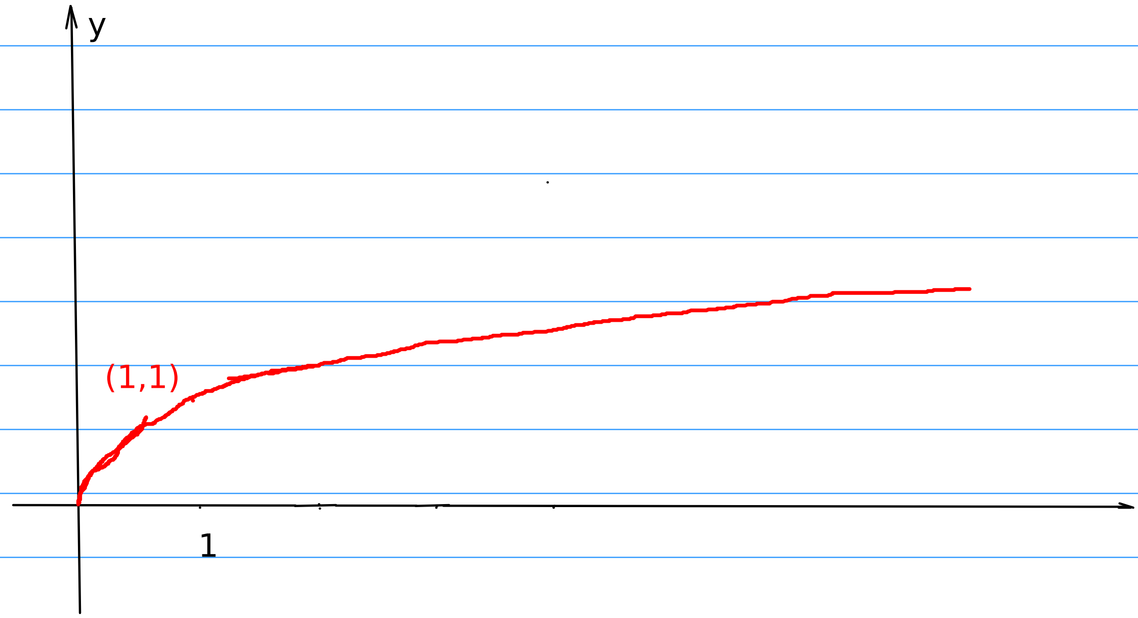
The entire graph would be shifted down 2 units from the red curve.

Example Graph y = x^3 and y = (x-2)^3



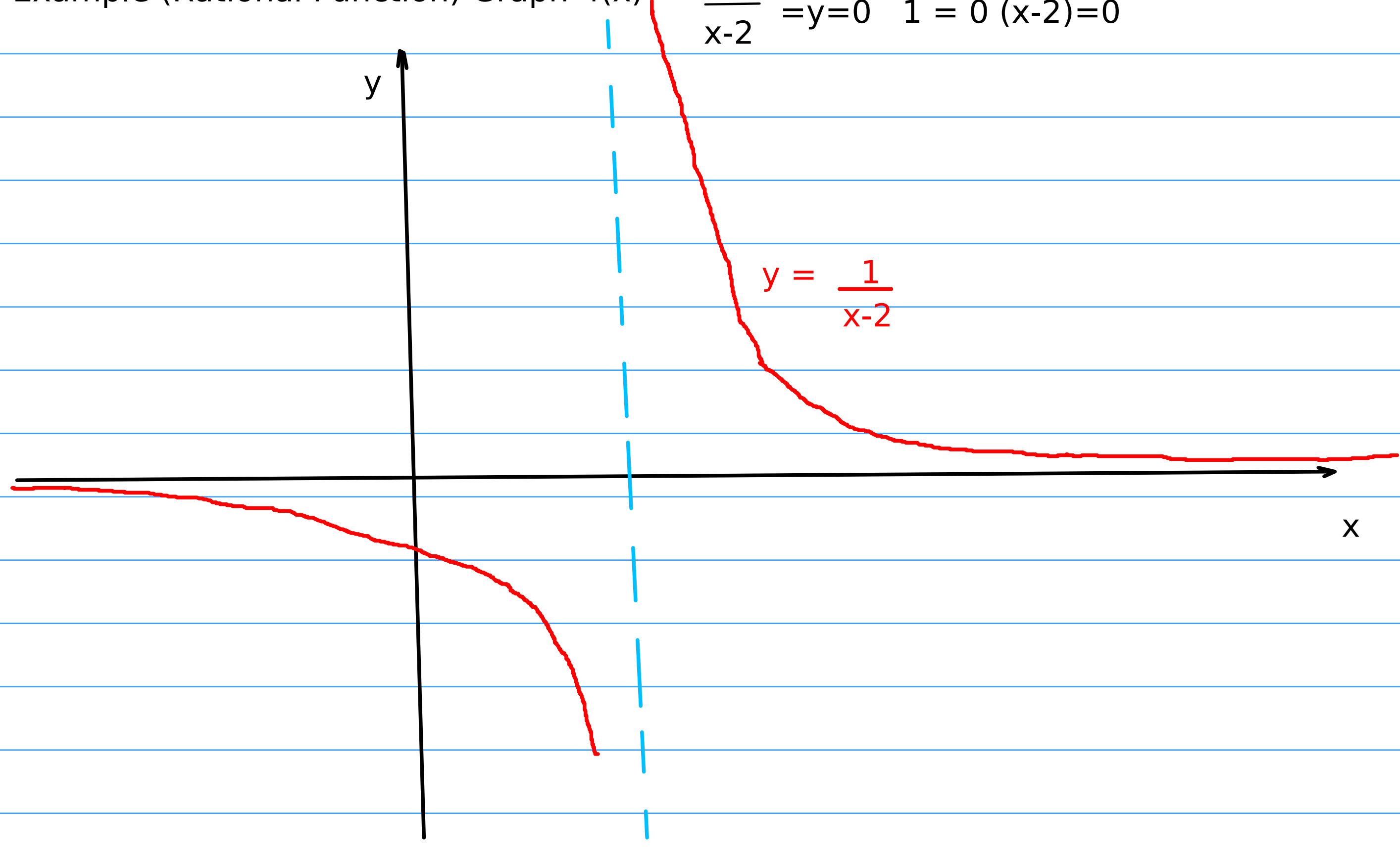
Example Graph y = $\sqrt{x}$ = x^(1/2)

This is the same as the curve x = y^2, y ⩾0 : half a parabola.



A rational function is the ratio of two polynomials.

Example (Rational Function) Graph f(x) = $\frac{1}{x-2}$ =y=0 1 = 0 (x-2)=0



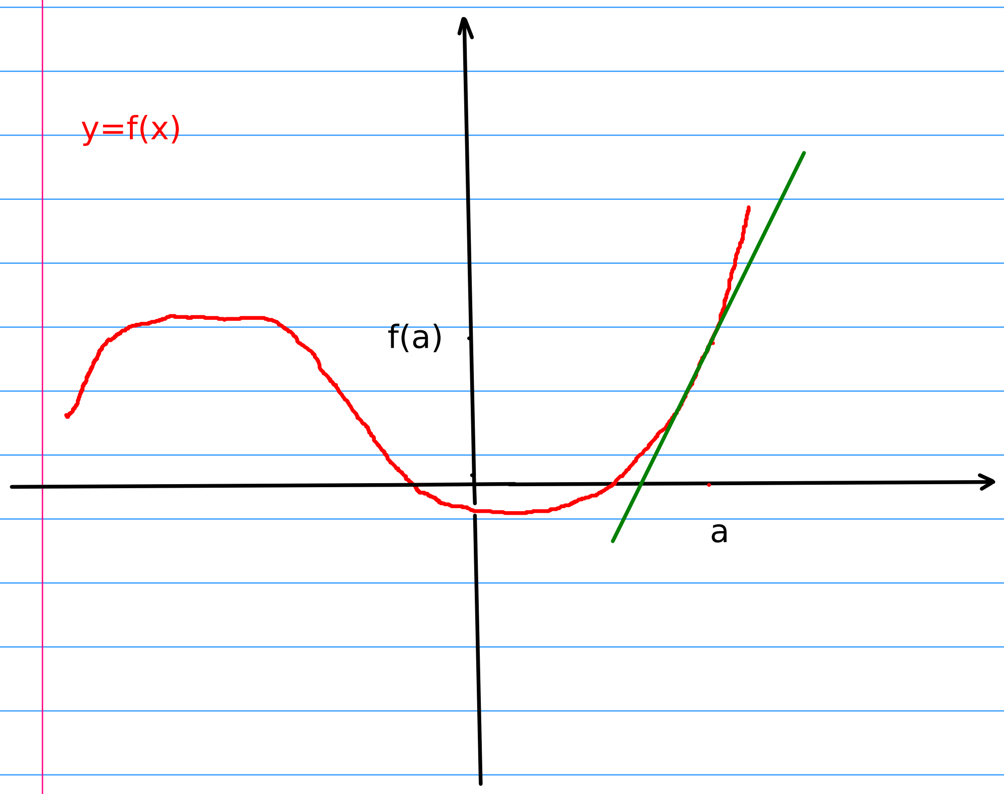
vertical asymptote x=2

Please read the example in Section R.5 especially Example 5 on page 57

Please read Section R.6 about modeling. You will learn a little about how functions arise in practical settings, that is outside the classroom.

1.1 Limits: A Numerical and Graphical Approach.

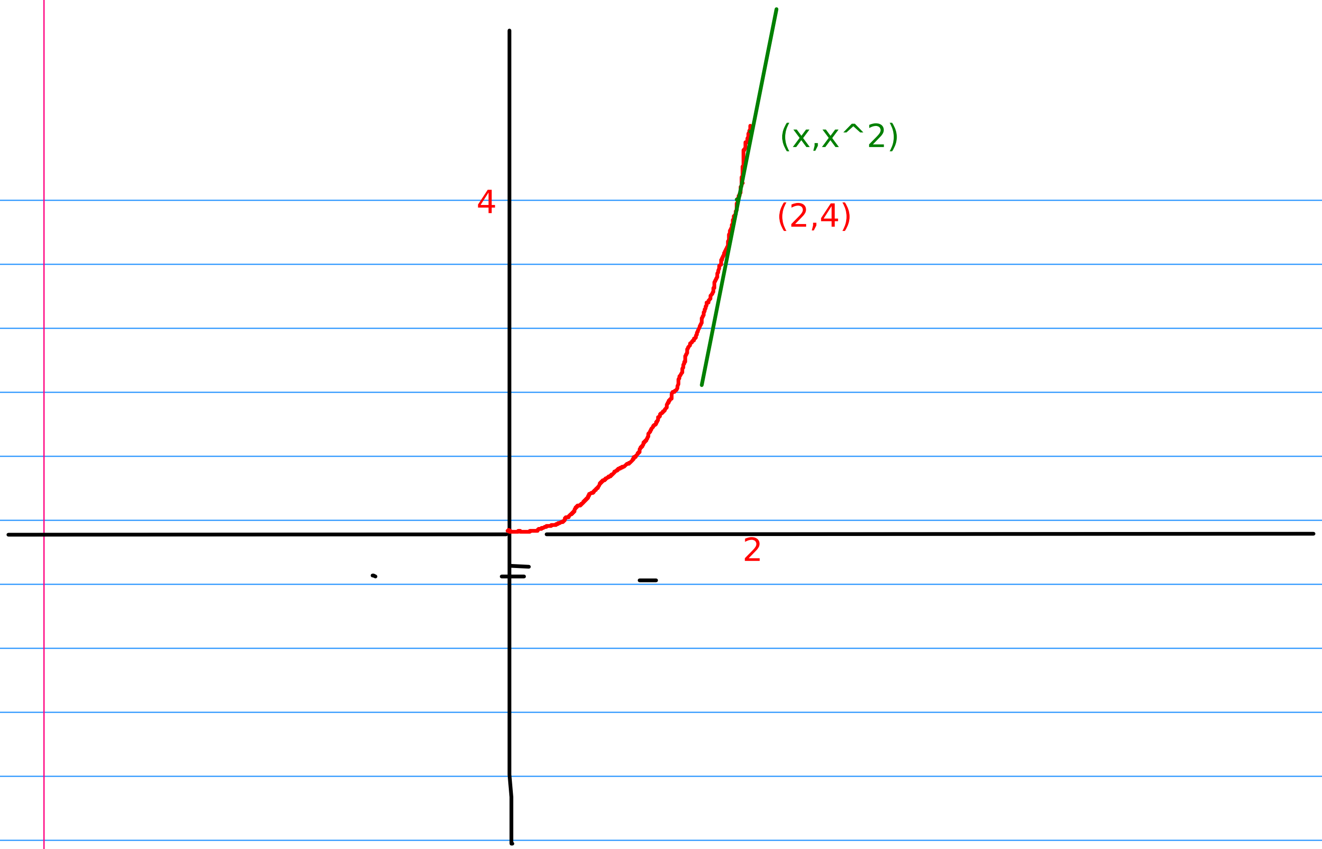
Orientation: We begin our study of calculus. We shall introduce ``differential calculus'' by talking about the slope of the graph of a function. We already know about the slope of lines and this is a generalization to curves. The slope of a graph y = f(x) at a point (a,f(a)) on the graph is the slope of the tangent line. What is the tangent line?



Example: f(x) = x^2 and a = 2. The tangent line at (2,4) must pass through (2,4) but we do not know another point on the line nor the slope. We approximate. Suppose $x \neq 2$ but x is near 2. Then (x,x^2) is on the curve and near (2,4). The slope of the ``secant'' line through (2,4) and (x,x^2) is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{x^2-4}{x-2}$$

x is near 2 or in the limit x is 2



limit as x approaches 2 of $\frac{x^2-4}{x-2}$ is we shall see 4

Ths slope of the tangent line is 4

Try x =2.1 x^2 = 4.41

$\frac{4.41-4}{2.1-2} = 4.1$