Syllabus

Summer Course is 8 weeks; Fall and Spring courses are 16 weeks.

Course online

Math 1730, June 10

Introduction: 1) Archimedes showed that the ratio of a volume of a sphere to the volume of its circumscribed cylinder is 2/3.

2) Isaac Newton used ''fluxions'' on his way to derive the laws of celestial mechanics and the Law of Universal Gravitation ($F = \frac{GMm}{r^2}$).

3) Gottfried Leibniz derived some properties of differentiation and integration (product rule and the fundamental theorem of ... calculus)

Subsequently it has been discovered that the easiest way to explain calculus is in terms of ''functions'' which should be familiar to everyone from earlier courses.

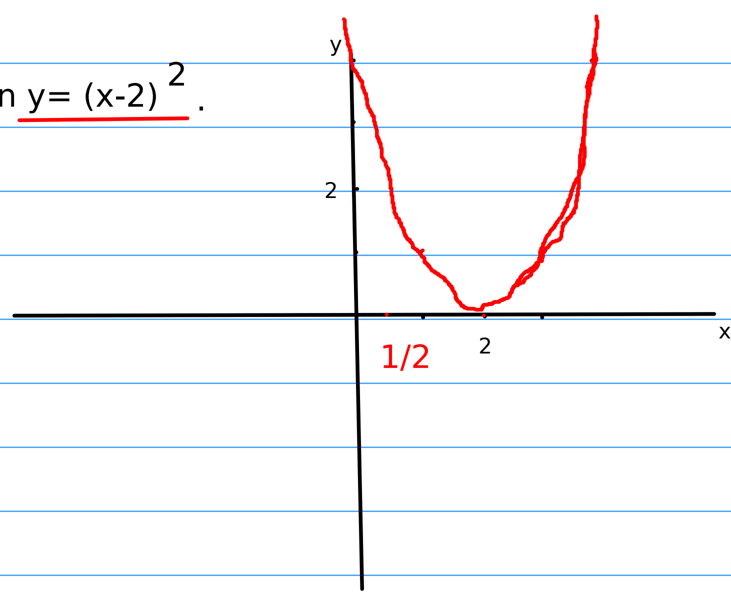
Chapter R recalls the properties of functions that will be needed in our study of calculus which starts in Chapter 1.

Chapter R.1. Graphs and Equations

Example. Graph the equation $y= (x-2)^2$.

(that is $y=x^2 -4x+4$)

x	y
2	0
1	1
3	1
0	4



Compound Interest Formula

If P dollars are deposited to a bank account that earns interest at the nominal rate of i compounded n times in a year then after t years the account is worth

A = P(1+ i/n)^(nt)   (Text page 9, Theorem 2)

Example: An account earns 3 percent compounded monthly. If 2000 is deposited today then the account is worth, after 5 years

$$2000(1.0025)^{60} = 2323.23$$

$$i/n = 0.03/12 = .0025$$

$$nt = 12*5 = 60$$

R.2 Functions and Models

N={1,2,3,4,5, ...}

A set is a (well defined) collection of objects.

A function is a correspondance (relation) between each element of one set, called the domain, and a second set, called the range such that every member of the domain corresponds (is related) to one and only one element of the range.

Example: If the domain is some set of real numbers then the f(x) = x^2 assigns to every element x of the domain the value x^2 in the range (a subset of the nonnegative numbers)

Counterexample: g(x) = $\pm\sqrt{x}$ with domain any subset of the nonnegative real numbers. g(4) is ± 2 (the solutions of x^2=4) then g is not a functi on because it takes on more than one value.

Graph of a function. If f is a function with domain D then the graph of f is the curve in the xy plane given by {(x,y):x in D, y=f(x)}

Example: Graph f(x)=x^2 - 2x

Graph of $x^2 - 2x$

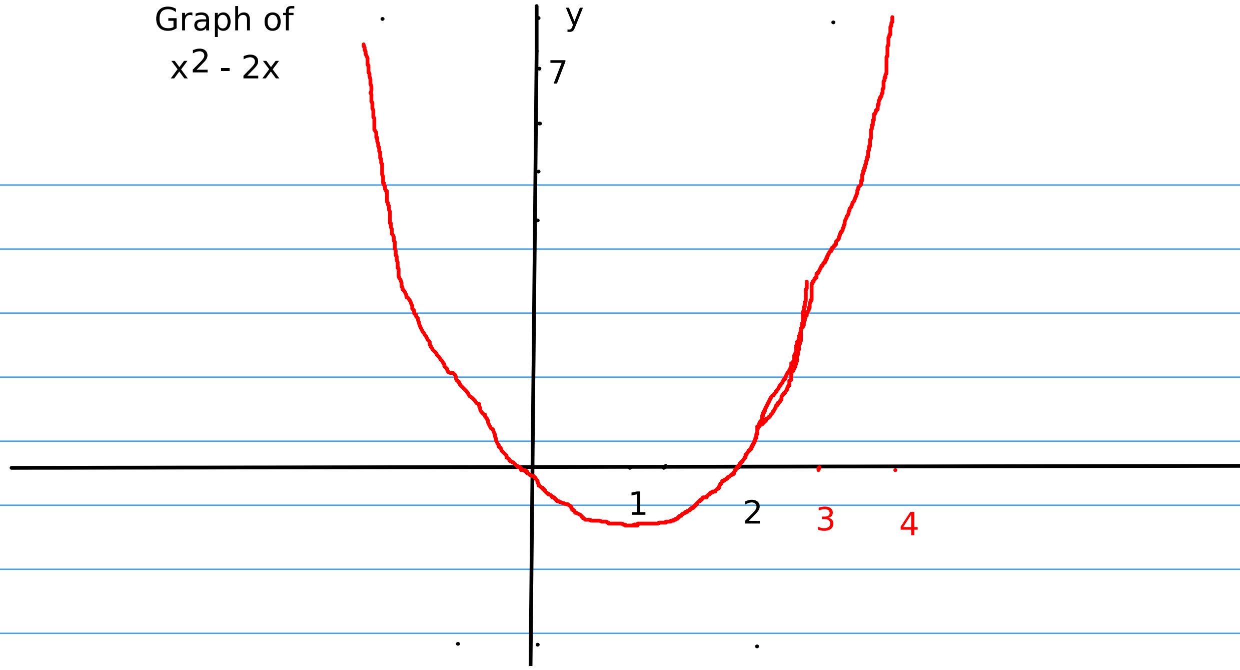
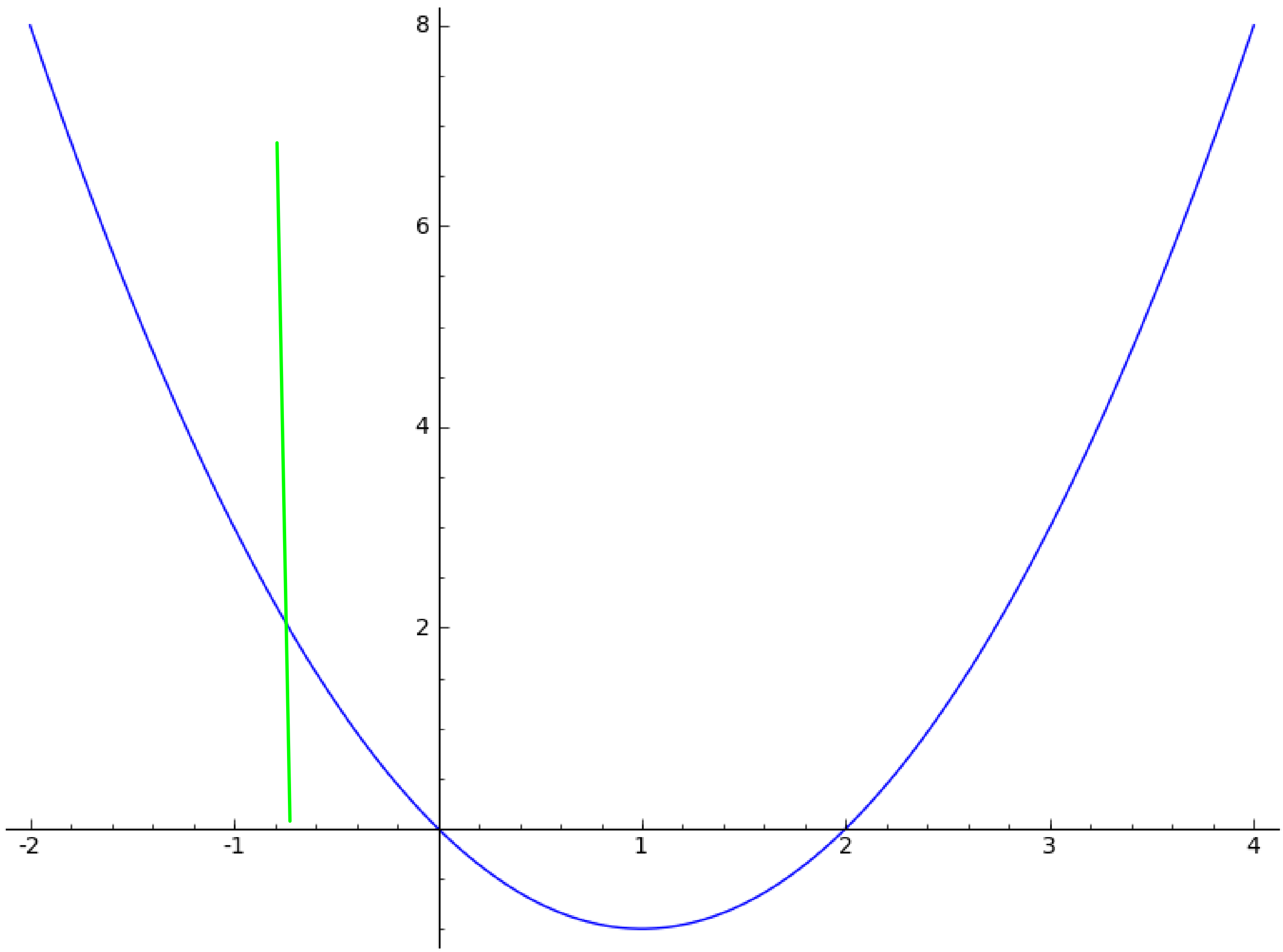


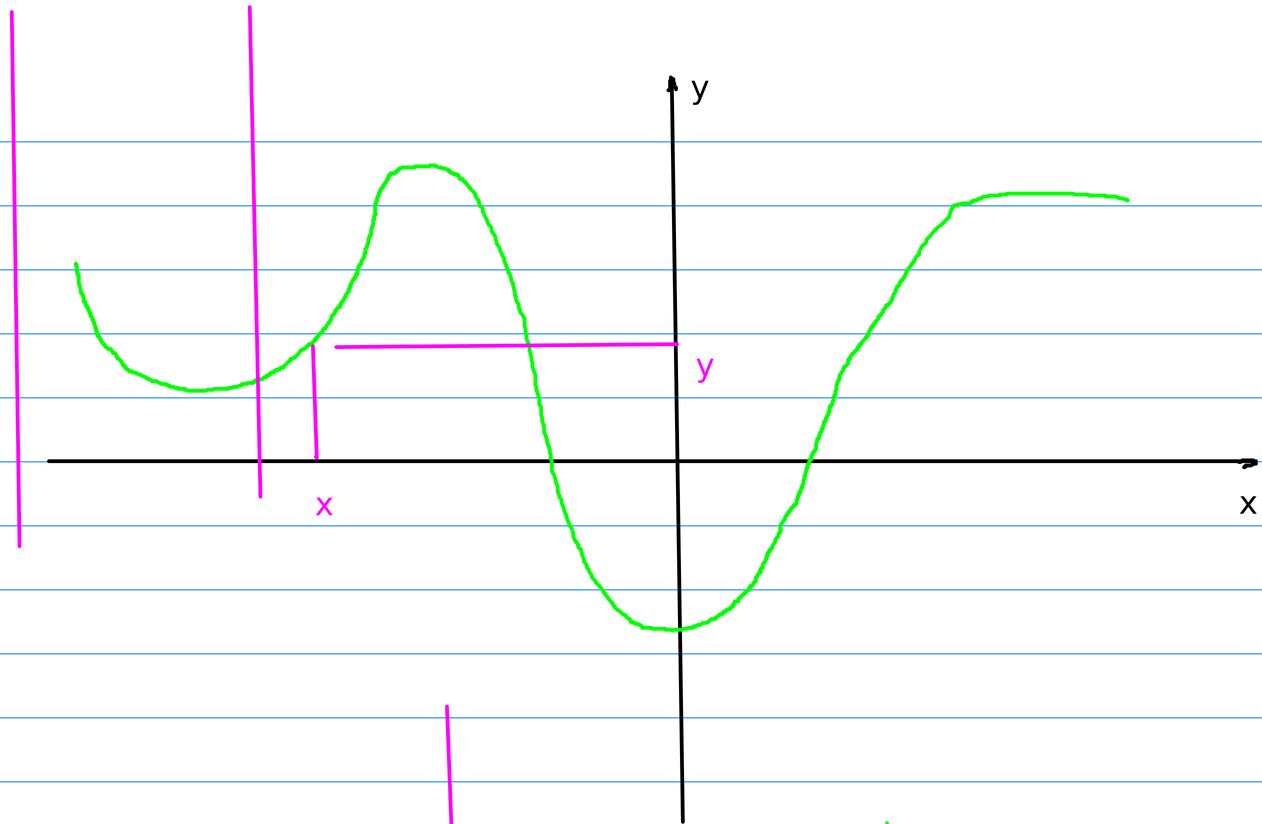
Table of Values

x	y
0	0
1	-1
2	0
3	3

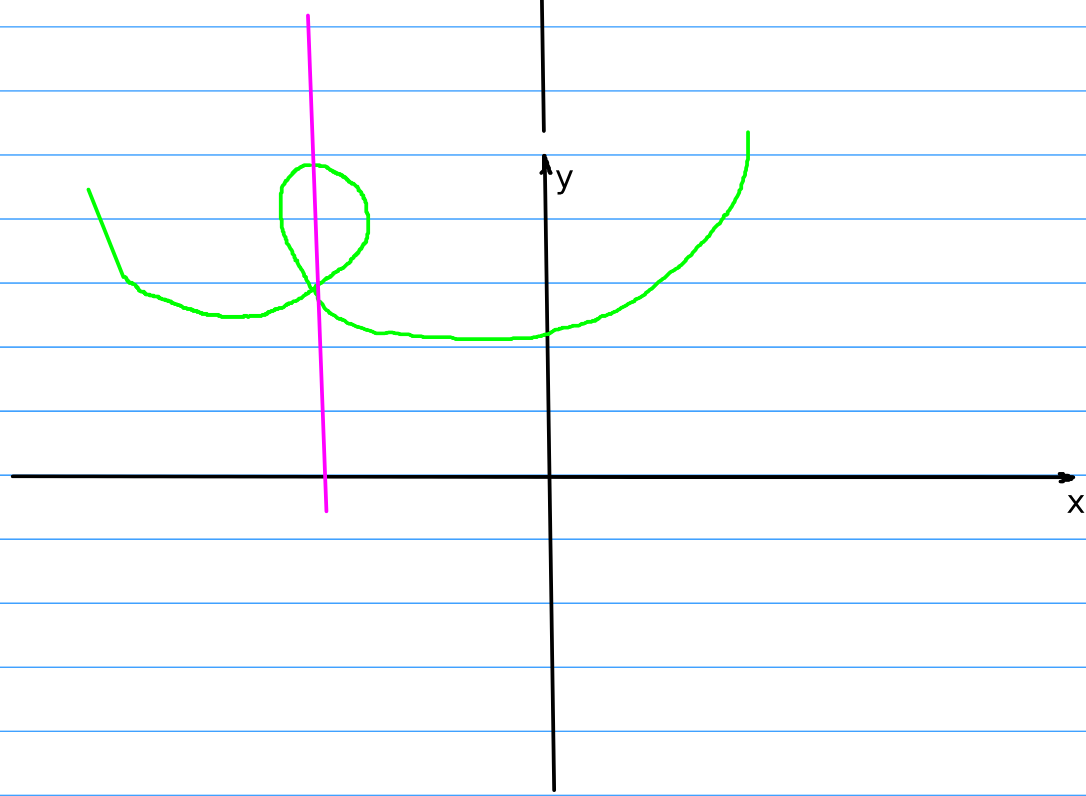


Each function with domain and range in the real numbers has a graph. Conversely if we draw a curve in the xy-plane then that curve is the graph of some function if and only if it obeys the vertical line test: A curve is the graph of a function if every vertical line intersects the graph at most once.

Example:



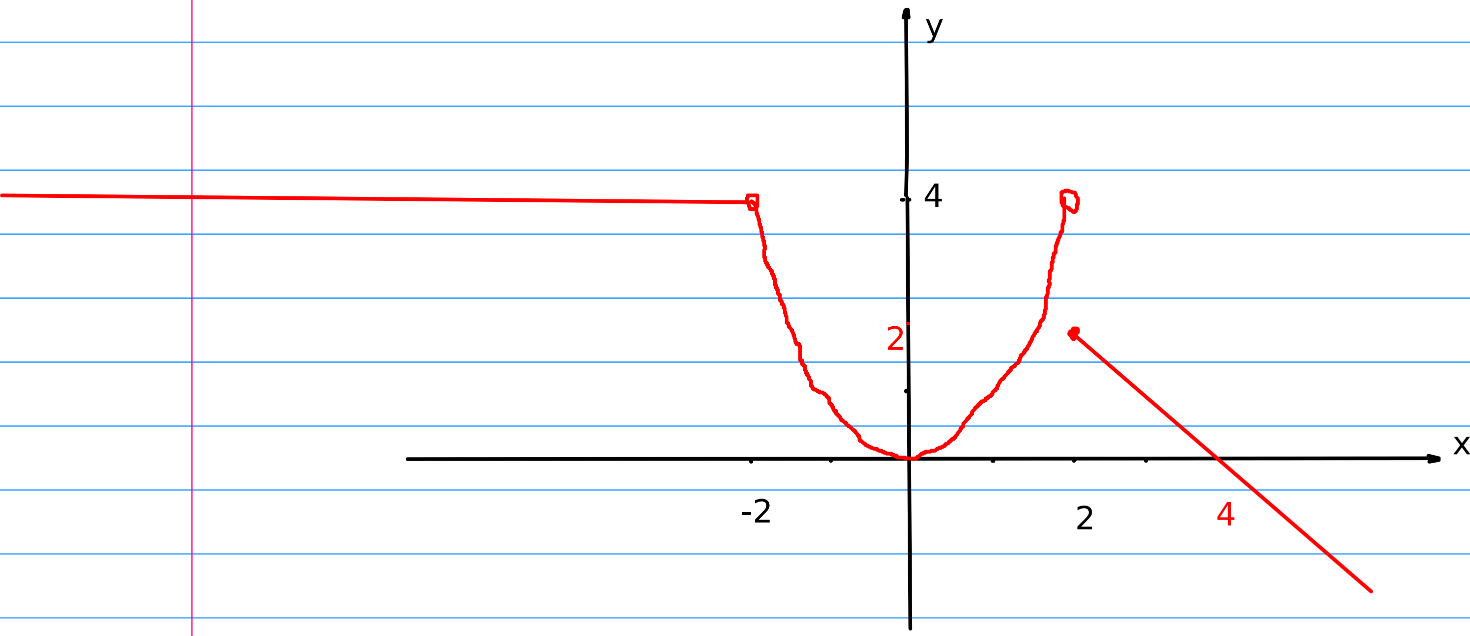
Counterexample



Functions Defined Piecewise:

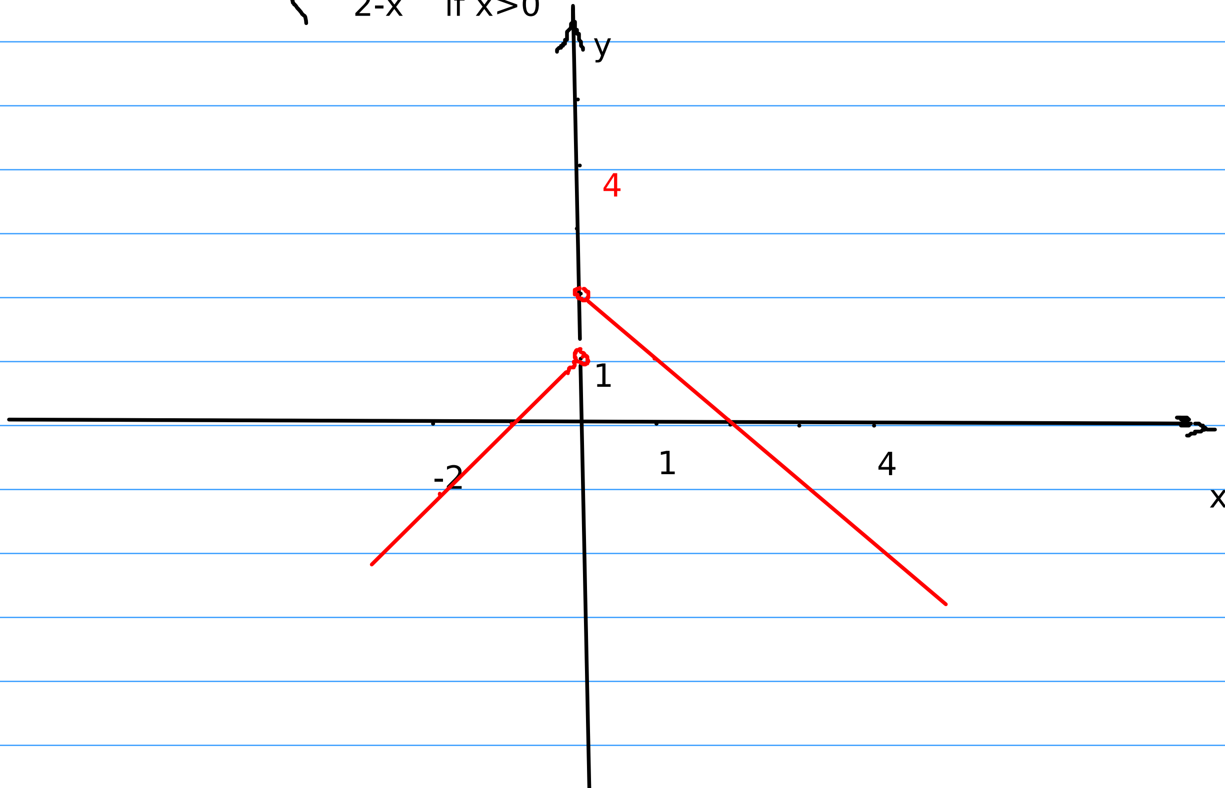
Example:

$$f(x) = \begin{cases} 4 & \text{if } x<-2 \\ x^2 & \text{if } -2 \le x < 2 \\ 4-x & \text{if } 2 \le x \end{cases}$$



Example:

$$f(x) = \begin{cases} x+1 & \text{if } x<0 \\ 2-x & \text{if } x>0 \end{cases}$$



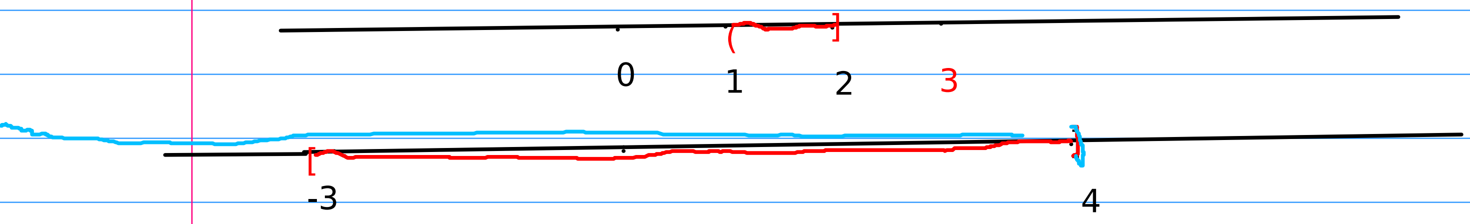
R.3 Domain and Range.

Interval Notation: $(1,2] = \{x: 1<x\leq 2\}$

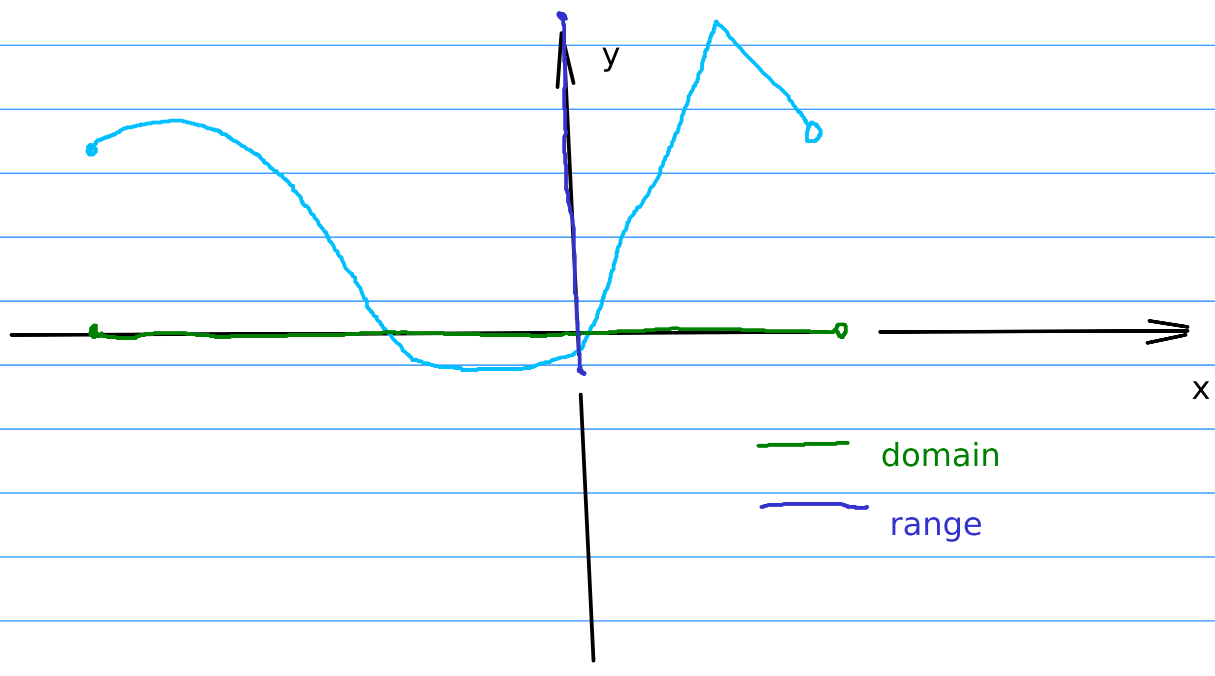
$[-3,4] = \{x \mid -3\leq x\leq 4\}$

$(-3,4)$

$(-\infty\ 4]$



Finding the domain and range of a function given its graph:



Finding the domain and range given a formula for the function.

Of course the domain might be specified, but if not then it is understood that the domain is a large as possible so that the formula for the function makes sense.

Example: $f(x) = \dfrac{4-2x}{x-3}$