Formulas Given on Final Exam

Consumer Surplus: $\int_0^Q D(x)dx - QP$

Producer Surplus: $QP - \int_0^Q S(x)dx$

Accumulated Future Value of an Income Stream: $A = e^{kT}\int_0^T R(t)e^{-kt}dt$

Accumulated Present Value of an Income Stream: $B = \int_0^T R(t)e^{-kt}dt$

Note:

- If you are asked to calculate a derivative using the definition of a derivative, you will be given the formula: $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

- Make sure you learn any other formulas that you need in order to solve the problems on the review sheet.
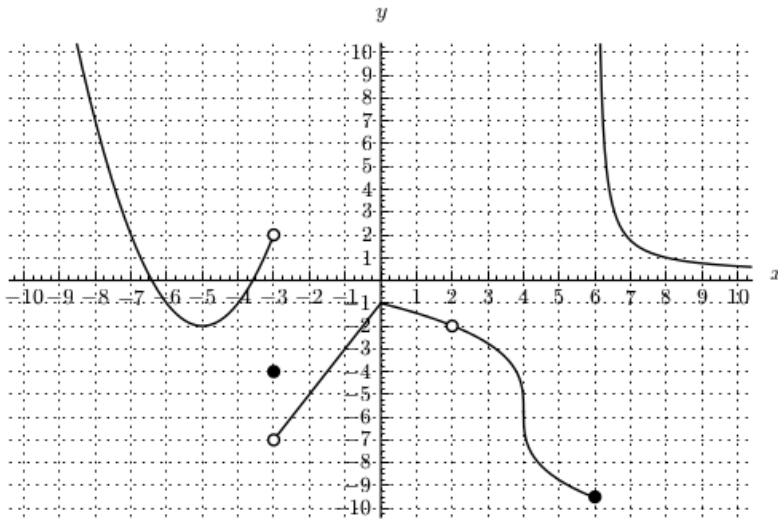
1. Given the above function find:

a. $\lim\limits_{x\to-3^-} f(x)=$ _____

b. $\lim\limits_{x\to-3^+} f(x)=$ _____

c. $\lim\limits_{x\to-3} f(x)=$ _____

d. $f(-3)=$ _____

e. $\lim\limits_{x\to 2^-} f(x)=$ _____

f. $\lim\limits_{x\to 2^+} f(x)=$ _____

g. $\lim\limits_{x\to 2} f(x)=$ _____

h. $f(2)=$ _____

i. $\lim\limits_{x\to 6^-} f(x)=$ _____

j. $\lim\limits_{x\to 6^+} f(x)=$ _____

k. $\lim\limits_{x\to 6} f(x)=$ _____

l. $f(6)=$ _____

m. $\lim\limits_{x\to 8^-} f(x)=$ _____

n. $\lim\limits_{x\to 8^+} f(x)=$ _____

o. $\lim\limits_{x\to 8} f(x)=$ _____

p. $\lim\limits_{x\to\infty} f(x)=$ _____

2. At what points is the graph above discontinuous?

3. For the graph above, at what values of x does the derivative not exist?

4. Find each of the given limits:

a. $\lim\limits_{x\to 2}\left(3x^3-7x^2+3\right)$

b. $\lim\limits_{x\to 3}\dfrac{x^2-9}{x-3}$

c. $\lim\limits_{x\to 1}\dfrac{x^2+3x-4}{x^2-1}$

5. Are each of the functions continuous at the given point or over the given interval?

a. $f(x)=\begin{cases}3x-5 & \text{for } x\le 2\\ x+1 & \text{for } x>2\end{cases}$ at the point $x = 2$

b. $f(x)=\begin{cases}x^2+4x+5 & \text{for } x\ne-3\\ 2 & \text{for } x=-3\end{cases}$ at the point x = -3

c. $g(x)=x^2-4x+2$ over the interval (-7, 7)

6. For $f(x)=-2x^2+3x-1$, a. find $f'(x)$ by determining $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

b. find $f'(-2)$ and $f'(3)$

7. Calculate the following derivatives:

a. $f(x)=x^3+4x^2-7x+5$

b. $f(x)=4x^{-2}+3x^{-1}$

c. $f(x)=\dfrac{5}{x^3}-\dfrac{4}{x^2}+\dfrac{6}{x}+\dfrac{x}{3}$

d. $f(x)=4x^{1/2}+7x^{3/2}$

e. $f(x)=\sqrt[3]{x^2}$

f. $f(x)=5x^{0.7}+3x^{0.2}$

8. Find the equation of the tangent line to the graph of $f(x)=x^3-2x^2+5x$ at the point (2, 10).

9. The average price in dollars of a share of a stock can be estimated by $p(x) = 9 - 0.2x + 0.1x^2$ where x is the number of years after the year 2000. Find:

    a. the rate of change of the average share price with respect to the year, dp/dx.

    b. The average share price in 2015.

    c. The rate of change of the average share price in 2015.

10. Use the product rule to calculate the derivative of $f(x) = (x^2 - 3x)(4x - 7)$.

11. Use the quotient rule to calculate the derivative of $f(x) = \dfrac{x^3 - 2x}{5 - x^2}$

12. Calculate the derivatives of each of the following:

    a. $f(x) = 3(x^2 - 5x - 12)^4$ b. $f(x) = 4\sqrt{6x^2 - 7x}$

13. Calculate the second derivative of each of the following:

    a. $f(x) = 7x^4 - 2x^2 + 9$ b. $f(x) = 5(4x^2 - 7)^3$

14. Calculate $f^{(4)}(x)$ given that $f(x) = e^{2x}$

Chapter 2

15. Find the relative extrema (maximums and minimums) of $f(x) = x^3 + 3x^2 - 24x + 10$.

16. Given $f(x) = x^3 - 2x^2 - 4x + 3$ find:

    a. The critical values

    b. The x values where any relative extrema occur

    c. The x values where any points of inflection occur

    d. Region(s) in which the function is increasing

    e. Region(s) in which the function is decreasing

    f. Region(s) in which the function is concave up

    g. Region(s) in which the function is concave down

17. Find the horizontal asymptotes (use limits) and vertical asymptotes for each of the following functions:

    a. $f(x) = \dfrac{x^2 + 5x + 6}{x^2 - 9}$ b. $f(x) = \dfrac{-2x}{x^2 - 1}$

18. Find the following limits:

    a. $\lim\limits_{x \to \infty} \dfrac{3x^2 - 5x + 1}{4 - x^2}$ b. $\lim\limits_{x \to \infty} \dfrac{2x^2 - 4x}{x + 5}$

19. Find the absolute maximum and absolute minimum for the function $f(x) = 12 + 9x - 3x^2 - x^3$ over [0, 3]

20. Find the maximum profit and the number of units that must be produced and sold in order to yield the maximum profit. (assume R(x) and C(x) are in dollars).
$$R(x) = 2x$$
$$C(x) = .01x^2 + 0.6x + 30$$

21. A store can sell 20 bicycles per week at a price of $400 each. The manager estimates that for each $10 price reduction she can sell two more bicycles per week. The bicycles cost the store $200 each. Find the price that will maximize the manager's annual profit. Also state what her maximum profit will be.

22. A company finds that its monthly revenue, in dollars, from the sale of x leather bags is $R(x) = .006x^3 - 0.4x^2 + 130x$ and the cost is $C(x) = 40x + 100$. Currently, the company is selling 32 bags per month.
   a. What is the marginal profit function?
   b. What is the marginal profit when 32 bags are sold?

Chapter 3

23. Calculate the following derivatives:
   a. $f(x) = 7e^{5x}$
   b. $f(x) = x^3 + e^{-3x}$
   c. $f(x) = 2e^{x^3 + 2x}$

   d. $f(x) = \ln(x^2 - 5x + 6)$
   e. $f(x) = x^2 \ln(3x)$
   f. $f(x) = \dfrac{e^{4x}}{x^4}$

24. If an initial deposit is made of $12,000 in an account paying 4% interest compounded continuously.
   a. How much will the account be worth in 6 years?
   b. How long will it take the account to double?

25. Find the present value of a payment of $2300 that will be received 3 years for now, assuming that interest is 2.7% compounded continuously.

26. Calculate the following derivatives:
   a. $f(x) = 4^x$
   b. $f(x) = \log_9 x$
   c. $f(x) = 7^{2x^2 - 5x + 7}$
   d. $f(x) = 5\log_3(x^4 - 7x)$

Chapter 4

27. Find the following indefinite integrals:
a. $\int (15x^2 + 5x - 7)dx$
b. $\int \left(\frac{4}{x^2} - \frac{7}{x}\right) dx$
c. $\int (7e^{2x} + 7x^2)dx$
d. $\int (\sqrt[3]{x^2} - \sqrt{x}) dx$

28. Find f such that $f'(x) = 6x^2 + 8x - 4$ and $f(1) = 10$.

29. Evaluate the following definite integral: $\int_{-2}^{6}(-x^2 + 4x + 14)dx$

30. A technology firm finds that the marginal profit, in dollars, from sale of x hard drives is given by $P'(x) = 3x^{0.5}$ A customer orders 50 hard drives and later increases the order to 60. Find the extra profit resulting from the increase in order size.

31. Find the area under the curve over the interval given:
a. $f(x) = 81 - x^2$; [-9, 9]
b. $f(x) = e^{2x}$; [0, 3]   (round answer to 3 decimal places)

32. Find the area of the regions bounded by the graphs of
$$y = x^2 + 2, y = x^2, x = -1, \text{ and } x = 2$$

33. Given the graphs of $y = x^2 + 2$ and $y = 3x + 6$.
    a. At what x value(s) do the graphs intersect?
    b. Find the area of the region bounded by the two functions.

34. Find the average value of the function $f(x) = x^2 + 2x + 10$ over the interval [-2, 6].

35. Use substitution to calculate the following integrals:
    a. $\int \frac{1}{2x+5} dx$ b. $\int x^2 e^{x^3} dx$ c. $\int_1^2 x(x^2 - 1)^7 dx$

Chapter 5

36. The price, in dollars per unit, that consumers are willing to pay for x units of an item, is given by $D(x) = (x - 5)^2$. The price in dollars per unit, that producers are willing to accept for x units of an item is given by $S(x) = x^2 + 2x + 13$. Find:
    a. The coordinates of the equilibrium point.
    b. The Consumer Surplus.
    c. The Producer Surplus.

37. A man receives a continuous income stream of $520,000 per year. If the income stream is invested at 6% compounded continuously for 25 years find (round answers to the nearest dollar):
    a. The accumulated future value of the income stream.
    b. The accumulated present value of the income stream.

38. Determine whether each improper integral is convergent or divergent, and calculate its value if it is convergent.
a. $\int_2^\infty \frac{dx}{x^3}$ b. $\int_0^\infty e^{2x} dx$ c. $\int_0^\infty 3e^{-3x} dx$

39. Find the accumulated present value of an investment for which there is a perpetual continuous money flow of $3900 per year, assuming continuously compounded interest at a rate of 8%.

Solutions

1.

a.	2	e.	-2	i.	-9.5	m.	1
b.	-7	f.	-2	j.	∞	n.	1
c.	DNE	g.	-2	k.	DNE	o.	1
d.	-4	h.	undefined	l.	-9.5	p.	0

2. -3, 2, 6

3. -3, 0, 2, 4, 6

4. a. -1   b. 6   c. 5/2

5. a. no   b. yes   c. yes

6. $-4x+3$, 11, -9

7. a. $3x^2 + 8x - 7$ b. $-8x^{-3} - 3x^{-2}$ c. $-15x^{-4} + 8x^{-3} - 6x^{-2} + \dfrac{1}{3}$ or $-\dfrac{15}{x^4} + \dfrac{8}{x^3} - \dfrac{6}{x^2} + \dfrac{1}{3}$

   d. $2x^{-1/2} + \dfrac{21}{2}x^{1/2}$ e. $\dfrac{2}{3}x^{-1/3}$ or $\dfrac{2}{3\sqrt[3]{x}}$ f. $3.5x^{-0.3} + 0.6x^{-0.8}$

8. $y = 9x - 8$

9. a. $\dfrac{dp}{dx} = -0.2 + 0.2x$ b. \$28.50   c. \$2.80/year

10. $(x^2 - 3x)4 + (2x - 3)(4x - 7)$ or $12x^2 - 38x + 21$

11. $\dfrac{(5 - x^2)(3x^2 - 2) - (x^3 - 2x)(-2x)}{(5 - x^2)^2}$ or $\dfrac{-x^4 + 13x^2 - 10}{(5 - x^2)^2}$

12a. $12(x^2 - 5x - 12)^3(2x - 5)$

12b. $2(6x^2 - 7x)^{-\frac{1}{2}}(12x - 7)$

13a. $84x^2 - 4$

13b. $1920x^2(4x^2 - 7) + 120(4x^2 - 7)^2$

14. $16e^{2x}$

15. Relative minimum at (2, -18); relative maximum at (-4, 90)

16a. x = -2/3, x = 2

16b. Relative maximum at x = -2/3, relative minimum at x = 2

16c. point of inflection at x = 2/3

16d. $\left(-\infty, -\dfrac{2}{3}\right), (2, \infty)$

16e. $\left(-\dfrac{2}{3}, 2\right)$

16f. $\left(\frac{2}{3}, \infty\right)$

16g. $\left(-\infty, \frac{2}{3}\right)$

17a. horizontal: y = 1, vertical: x = 3

17b. horizontal: y = 0, vertical: x = 1, x = -1

18a. -3

18b. ∞

19. Absolute maximum of 17 an x = 1
    Absolute minimum of – 15 at x = 3

20. x = 70, maximum profit is $19.00

21. $350 each, profit: $4500.

22a. $P'(x) = 0.018x^2 - 0.8x + 90$

22b. $82.83

23a. $f'(x) = 35e^{5x}$

23b. $f'(x) = 3x^2 - 3e^{-3x}$

23c. $f'(x) = 2e^{x^3+2x}(3x^2 + 2)$

23d. $f'(x) = \dfrac{(2x - 5)}{x^2 - 5x + 6}$

23e. $f'(x) = x + 2x\ln(3x)$

23f. $f'(x) = \dfrac{4x^4 e^{4x} - 4x^3 e^{4x}}{x^8}$ or $\dfrac{4xe^{4x} - 4e^{4x}}{x^5}$

24a. $15,254.99

24b. approximately 17 years

25. $2121.05

26a. $f'(x) = (\ln 4) \cdot 4^x$

26b. $f'(x) = \dfrac{1}{(\ln 9)x}$

26c. $f'(x) = (\ln 7) \cdot 7^{2x^2-5x+7}(4x - 5)$

26d. $f'(x) = \dfrac{5(4x^3 - 7)}{(\ln 3)(x^4 - 7x)}$

27a. $5x^3 + \left(\frac{5}{2}\right)x^2 - 7x + C$

27b. $-4x^{-1} - 7(\ln x) + C$

27c. $\left(\frac{7}{2}\right)e^{2x} + \left(\frac{7}{3}\right)x^3 + C$

27d. $\left(\frac{3}{5}\right)x^{\frac{5}{3}} - \left(\frac{2}{3}\right)x^{\frac{3}{2}} + C$

28. $f(x) = 2x^3 + 4x^2 - 4x + 8$

29. $\frac{304}{3}$ or $101\frac{1}{3}$

30. $222.41

31a. 972

31b. 201.214

32. 6

33a. -1,4

33b. 125/6

34. 70/3

35a. $\frac{1}{2}\ln(2x + 5) + C$

35b. $\frac{1}{3}e^{x^3} + C$

35c. 6561/16

36a. (1, $16)

36b. $4.33

36c. $1.67

37a. $30,174,639

37b. $6,732,872

38a. converges ,1/8

38b. diverges

38c. converges, 1

39. $48,750