Homework # 17- Due Tuesday 3/21/06, Assigned 3/14/06

1. Determine the radical and socle series of the $T_n(k)$-module $T_n(k)$.

2. If U is an A-module, prove the radical length and socle length are equal. (The common length is called the Loewy length of U)

3. Suppose U has Loewy length s. Prove that for $0 \leq i \leq s$ that:

$$\mathrm{rad}^i(U) \subseteq \mathrm{soc}^{s-i}(U).$$

4. Let A_0 be an algebra not necessarily with identity. Prove there is an algebra A with identity 1 containing A_0 as a subalgebra of codimension one. Show that every ideal of A_0 is an ideal of A. Prove that if A_0 has no non-zero nilponent ideal then the same is true for A. Deduce that A_0 is a direct sum of matrix algebras. (And thus had an identity to begin with!)

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