Homework #3 Solutions

3-8. Image of 3 is 1. 4 & 5 are fixed, 1, 2, 3 are moved.

$$P^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 4 & 5 \end{pmatrix}$$

3-9

$$P \circ Q = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix} \quad Q \circ P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 5 & 2 \end{pmatrix}$$

$$Q^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & \cancel{4}3 & 2 & 4 \end{pmatrix}$$

3-10 $$Q^{-1} \circ P^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix}$$

$$(P \circ Q) \circ (Q^{-1} \circ P^{-1}) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$ by assoc. law.

3-14 $$(134) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$$

$$(12)(34) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$$(1423) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$$

$$(13)(12) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$$

$$(14)(13)(12) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

3. Suppose $x \in G$ appears twice in column corresponding to g:

$$\begin{array}{c|c} & g \\ \hline g_1 & x \\ g_2 & x \end{array}$$

Then $g_1 g = g_2 g = x$ for some $g_1, g_2 \in G$.

Then $g_1 = g_2$ by cancellation. Thus x appears in at most one. However

$(xg^{-1})g = x$ so in

column g, x appears in the row of xg^{-1} so it appears exactly once in each column

4. Suppose $g^2 = e$ $\forall g \in G$. Let $a, b \in G$. Then

$(ab)^2 = e$ so $abab = e$.

$a^2bab = a$ mult on left by a.

$bab = a$ $\quad a^2 = e$

$b^2ab = ba$

$ab = ba$ $\quad b^2 = e$

Thus $ab = ba$ $\forall a, b \in G$ so G is abelian.

5: Let $|G| = m$, $g \in G$.

$\{e, g, g^2, \ldots, g^m\}$ has $m+1$ elements in it,

so there must be at least one repeat, say $g^i = g^j$

for some $i < j$. Then $g^{j-i} = e$ by cancellation,

and $j - i > 0$.

HW#4 SOLUTIONS

3-15

Markers start: $DCEBA$.

One way to alphabetize:

$$DCEBA \xrightarrow{(AD)} ACEBD \xrightarrow{(BC)} ABECD \xrightarrow{(CE)} ABCED \xrightarrow{(DE)} ABCDE$$

This took 4 transpositions. Thus any solution must use an even # of moves by Thm 3-4. Thus let your opponent go first, you must win!

2. On HW #3 Problem 5 we proved that $g^m = e$ for some $m \geq 1$. Thus $g^{-1} = g^{m-1}$ is a power of g.

Example $G = \mathbb{Q}^*$ under mult.

$$g = \tfrac{1}{2} \quad g^2 = \tfrac{1}{4} \quad g^3 = \tfrac{1}{8} \quad \ldots$$

$g^{-1} = 2$ is not some positive power of g.

3

σ	order
(12345)	5
$(123)(45)$	6
$(123)(456)(78)$	6
$(12)(34)(56)$	2
$(123)(456789)$	6
$(1234567)(8910)(111213141516)(1712)$	42

RMK When σ is in disjoint cycle notation the order is the lcm of the cycle lengths.

4.

	e	$(12)(34)$	$(13)(24)$	$(14)(23)$
e	e	$(12)(34)$	$(13)(24)$	$(14)(23)$
$(12)(34)$	$(12)(34)$	e	$(14)(23)$	$(13)(24)$
$(13)(24)$	$(13)(24)$	$(14)(23)$	e	$(12)(34)$
$(14)(23)$	$(14)(23)$	$(13)(24)$	$(12)(34)$	e

5.

	e	(12)	(34)	$(12)(34)$
e	e	(12)	(34)	$(12)(34)$
(12)	(12)	e	$(12)(34)$	(34)
(34)	(34)	$(12)(34)$	e	(12)
$(12)(34)$	$(12)(34)$	(34)	(12)	e

	e	(1234)	$(13)(24)$	(1432)
e	e	(1234)	$(13)(24)$	(1432)
(1234)	(1234)	$(13)(24)$	(1432)	e
$(13)(24)$	$(13)(24)$	(1432)	e	(1234)
(1432)	(1432)	e	(1234)	$(13)(24)$

6. $\sigma = (1234)(567)(89) = (14)(13)(12)(57)(56)(89)$

even

$\tau = (1234567) = (17)(16)(15)(14)(13)(12)$ even

7. $(12)(234)(34)(15) = (152\ 3)(4)$

$(1234)(14)(23)(1432) = (12)(34)$

$(15)(14)(13)(12) = (12345)$