

Homework # 9- Due Tuesday 2/14/06, Assigned Tuesday 2/7/06

1. Let G be abelian and let $H = \{g \in G \mid g^2 = e\}$. Prove that $H \leq G$. Give an example that demonstrates the hypothesis that G is abelian is necessary.
2. Let G be a group and let a be one fixed element of G . Show that:

$$H_a = \{x \in G \mid xa = ax\}$$

is a subgroup of G .

3. Show that S_n is nonabelian for any $n \geq 3$.
4. Give an example of a group G which has no elements of finite order > 1 but has a factor group G/H , all of whose elements have finite order.
5. Let $H \leq G$ and fix $g \in G$. Define:

$$gHg^{-1} = \{ghg^{-1} \mid h \in H\}.$$

- a. Prove that $gHg^{-1} \leq G$. It is called a *conjugate* of H .
- b. Suppose G is finite. Prove that $|gHg^{-1}| = |H|$
- c. Now suppose G is a group and H is the unique subgroup of size $|H|$ in G . Prove that $H \trianglelefteq G$.

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