

**Homework # 2- Due Tuesday January 17, Assigned Thursday January 12**

1. Definitions: abelian, Klein 4-group, dihedral group
2. Exercises 1-15, 1-17
3. Let  $G$  be a finite group with an *even* number of elements. Prove that there is at least one element  $g$  other than the identity which is its own inverse, i.e. such that  $g * g = e$ .
4. Consider the set of integers  $\{0, 1, 2, \dots, 7\}$  with the binary operation given by addition *modulo 8*, (e.g.  $5 + 4 \equiv 1$ ). Write down the addition table for this binary operation. Is it a group? If so, what is the inverse of each element. If not, what property fails
5. Now consider the set of integers  $\{1, 2, \dots, 7\}$  where the operation is *multiplication modulo 8*. Write down the table. Is this a binary operation? If so is it a group?
6. Repeat exercise 4 for the integers  $\{0, 1, 2, \dots, 6\}$  with addition modulo 7.
7. Repeat exercise 5 for the integers  $\{1, 2, \dots, 6\}$  with multiplication modulo 7.
8. Bonus: Can you conjecture any theorem based on exercises 5 and 7?