

Homework # 18- Due Tuesday 4/11/06, Assigned 4/6/06

1. Book 7-12, 7-14.
2. Find the minimum polynomials over the small field of the following elements in the following extensions:
 - a. i in $\mathbb{C} : \mathbb{Q}$
 - b. $\sqrt[3]{7}$ in $\mathbb{R} : \mathbb{Q}$
 - c. $\frac{1+\sqrt{5}}{2}$ in $\mathbb{C} : \mathbb{Q}$
 - d. $\sqrt{5} + \sqrt{3}$ in $\mathbb{C} : \mathbb{Q}$
3. Recall that $\mathbb{Z}_3 = \{0, 1, 2\}$ is a field with three elements under operations modulo three.
 - a. Find an irreducible polynomial $p(x) \in \mathbb{Z}_3[x]$ of degree 2. Explain how you know it is irreducible.
 - b. Construct the field $E = \mathbb{Z}_3[x] / \langle p(x) \rangle$, i.e. give the multiplication and addition tables. There should be 9 elements in this new field.
 - c. Factor $p(x)$ into two linear terms in $E[x]$.
 - d. Bonus: Find the addition and multiplication tables for a field with 8 elements by starting with \mathbb{Z}_2 and an irreducible *cubic*.