Homework # 16- Due Tuesday 3/28/06, Assigned 3/21/06

1. Find all prime numbers p such that $x+2$ is a factor of $x^4+x^3+x^2-x+1$ in $\mathbb{Z}_p[x]$.

2. Find all irreducible polynomials of degree 3 in $\mathbb{Z}_3[x]$.

3. Demonstrate that $x^4 - 22x^2 + 1$ is irreducible over $\mathbb{Q}$. Be sure to justify your arguments.

4. Let $\phi : G \to H$ be a group homomorphism which is onto. Suppose G has 60 elements and H has 4 elements. Find $\mid \ker\phi \mid$. Let $h \in H$. How many elements of G map to h? (i.e. find the size of $\phi^{-1}(h)$) You map cite results from any of your previous HW problems.

5. Show that $\alpha = \sqrt{2} + \sqrt{3}$ is algebraic over $\mathbb{Q}$ by finding $f(x) \in \mathbb{Q}[x]$ such that $f(\alpha) = 0$. Repeat for

$$\alpha = \sqrt{\sqrt[3]{2} - i}.$$

6. True or False (Not to be handed in)

____ a. The intersection of two normal subgroups of G is also normal..
____ b. There are no abelian simple groups of order 15.
____ c. $\mathbb{C}$ is an extension field of $\mathbb{R}$.

____ d. $3x^3 + 6x^3 + 12x + 24$ is irreducible over $\mathbb{Q}[x]$ by the Eisenstein criterion.
____ e. The division algorithm works in the ring $\mathbb{Z}_{12}[x]$.
____ f. If $f(x) \in F[x]$ has a root in F then it cannot be irreducible over F.
____ g. If $f(x)$ is reducible in $F[x]$ then it must have a root in F.
____ h. Let $f : G \to H$ be a group homomorphism. If $g \in G$ and $g^4 = e$ then $f(g)^4 = e$.
____ i. Let $f : G \to H$ be a group homomorphism. If $g \in G$ has order 4 then $f(g)$ has order 4.
____ j. If $3x^3 + 2x^2 - 5x + 15$ has a rational root then it must be have an integer root which divides 15.
____ k. Let G be a group with 60 elements. Then G must have an element of order 4.
____ l. Let G be a nonabelian simple group. Then the center of G contains only one element.
____ m. Let $n = ab$ with $a, b \in \mathbb{Z}$. If $6 \mid ab$ then $6 \mid a$ or $6 \mid b$.
____ n. $x^2 + 1$ is irreducible over $\mathbb{C}$.
____ o. A_4 is a simple group.
____ p. It is possible to have a homomorphism from a group with 60 elements to a group with 25 elements which is onto.
____ q. If R has zero divisors then so does $R[x]$.
____ r. And cubic polynomial in $\mathbb{Q}[x]$ must have a root in $\mathbb{Q}$.