Homework # 14- Due Tuesday 3/21/06, Assigned 3/14/06

0. Read Chapter 6. Vocabulary: polynomial ring, coefficients, degree, leading term, division algorithm, root, irreducible, greatest common divisor

1. Find the sum and the product of the given polynomials in the given polynomial ring. Recall that $\mathbb{Z}_n$ denotes the ring of integers modulo n.
   a. $f(x) = 4x - 5, g(x) = 2x^2 - 4x + 2$ in $\mathbb{Z}_8[x]$.
   b. $f(x) = x + 1, g(x) = x + 1$ in $\mathbb{Z}_2[x]$.
   c. $f(x) = 2x^3 + 4x^2 + 3x + 2$, $g(x) = 3x^4 + 2x + 4$ in $\mathbb{Z}_5[x]$.

2. How many polynomials are there of degree ≤ 3 in $\mathbb{Z}_2[x]$? How many in $\mathbb{Z}_5[x]$? Include the zero polynomial.

3. Let $F = \mathbb{C}$. Calculate the following evaluation homomorphisms:
   a. $ev_2(3x^2 + 2x - 5)$
   b. $ev_{-1}(x^3 + 3x)$
   c. $ev_{-5}(6)$

4. Find all the roots in the indicated finite field of the given polynomial with coefficients in that field. Hint: One way is to simply try all candidates.
   a. $x^2 + 1$ in $\mathbb{Z}_2$
   b. $x^5 + 3x^3 + x^2 + 2x$ in $\mathbb{Z}_5$
   c $f(x)g(x)$ where $f(x) = x^3 + 2x^2 + 5$ and $g(x) = 3x^2 + 2x$ in $\mathbb{Z}_7$.

5. Prove that if R is an integral domain then $R[x]$ is also an integral domain. Hint: Look at the leading coefficients.

6. Exercises 6.18, 6.19, 6.23, 6.25 - 6.28