

Name: SOLUTIONS

Quiz #8 April 1, 2005

1. Use the Laplace transform to solve:

$$y^{(4)} - 4y = 0; \quad y(0) = 1, y'(0) = 0, y''(0) = -2, y'''(0) = 0.$$

$$s^4 \mathcal{L}(y) - s^3 y(0) - s^2 y'(0) - s y''(0) - y'''(0) = 0$$

$$s^4 \mathcal{L}(y) - s^3 + 2s = 0$$

$$\mathcal{L}(y) = \frac{s^3 - 2s}{s^4 - 4} = \frac{s^2 - 2}{s^3} = \frac{1}{s} - \frac{2}{s^3}$$

$$y = \frac{s(s^2 - 2)}{(s^2 + 2)(s^2 - 2)} = \frac{s}{s^2 + 2}$$

$$y = \cos(\sqrt{2} t)$$

2. Find the inverse Laplace transform of:

$$F(s) = \frac{e^{-4s}}{(s-2)^3}$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s-2)^3}\right) = \frac{1}{2} t^2 e^{2t}$$

$$\text{so } \mathcal{L}^{-1}\left(e^{-4s} \frac{1}{(s-2)^3}\right)$$

$$= \frac{1}{2} (t-4)^2 e^{2(t-4)}$$