p99

1. exact. $\psi = \int 2x+3\,dx = x^2+3x+g(y)$

$\psi = \int 2y-2\,dy = y^2-2y+h(x)$

$\psi = x^2+y^2+3x-2y$

solutions $x^2+y^2+3x-2y = c$

2. Not exact

4. exact $\psi = \int 2xy^2+2y\,dx = x^2y^2+2xy+h(y)$

$\psi = \int 2x^2y+2x\,dy = x^2y^2+2xy+g(x)$

$x^2y^2+2xy = c$

6. $ax-by+(bx-cy)\frac{dy}{dx} = 0$ Not exact

13. $(2x-y)dx+(2y-x)dy = 0 \quad y(1)=3$ This is exact

$\psi = \int 2x-y\,dx = x^2-xy+g(y)$

$\psi = \int 2y-x\,dy = y^2-xy$

$x^2+y^2-xy = C \quad x=1 \; y=3$

$1+9-3 = \cancel{a}\,7$

$x^2+y^2-xy = \cancel{8}\,7$

$y^2-xy+x^2-\cancel{8}7=0$

$y = \frac{x+\sqrt{x^2-4x^2+2\cancel{8}}}{2} = \frac{x+\sqrt{2\cancel{8}-3x^2}}{2} = y$

good for $-\sqrt{\frac{28}{3}} < x < \sqrt{28/3}$

25. $\mu(x,y)(3x^2y+2xy+y^3)\,dx+(x^2+y^2)\mu(x,y)\,dy=0$

Need $\mu_y(3x^2y+2xy+y^3)+\mu(3x^2+2x+3y^2)=2x\mu+(x^2+y^2)\mu_x$

Suppose μ is a function of x so $\mu_y=0$

$$\mu(3x^2+3y^2+2x)-2x\mu=(x^2+y^2)\mu_x$$

$$3(x^2+y^2)\mu=(x^2+y^2)\mu_x$$

$$3\mu=\mu_x$$

$$\mu=e^{3x}$$

$e^{3x}(3x^2y+2xy+y^3)+e^{3x}(x^2+y^2)=0$ This is exact

$\psi=\int e^{3x}(3x^2y+2xy+y^3)\,dx=x^2ye^{3x}+y^3e^{3x}\cdot\frac{1}{3}+h(y)$

$\psi=\int e^{3x}x^2+y^2e^{3x}\,dy=e^{3x}(x^2y+\frac{1}{3}y^3)$ *

$$e^{3x}(x^2y+\frac{1}{3}y^3)=C$$

27. $\mu(x)\,dx + \mu\left(\frac{x}{y} - \sin y\right)dy = 0$

Need $\mu_y = \mu_x\left(\frac{x}{y} - \sin y\right) + \mu \cdot \frac{1}{y}$

Assume $\mu_x = 0$

$\mu_y = \frac{1}{y}\mu$

$\mu = \cancel{\quad}$ $\qquad \frac{d\mu}{dy} = \frac{1}{y}\mu$

$\frac{1}{y}\,dy = \frac{1}{\mu}\,d\mu$

$\ln|y| = \ln|\mu| + C$

$y = \mu$

so $\boxed{\mu = y}$

$y\,dx + (x - y\sin y)\,dy = 0$

$\psi(x,y) = \int y\,dx = xy + g(y)$

$\psi(x,y) = \int x - y\sin y\,dy = xy + y\cos y - \sin y$

$xy + y\cos y - \sin y = C$

P.77

#28. $y' + \frac{2}{t}y = \frac{y^3}{t^2}$

Let $v = y^{-2}$ $\quad y = v^{-1/2}$ $\quad y' = -\frac{1}{2}v^{-3/2}v'$

$$-\frac{1}{2}v^{-3/2}v' + \frac{2}{t}v^{-1/2} = \frac{1}{t^2}v^{-3/2}$$

$\cdot\, v^{3/2}$

$$-\frac{1}{2}v' + \frac{2}{t}v = \frac{1}{t^2}$$

$$v' - \frac{4}{t}v = -\frac{2}{t^2}$$ Now it is linear, use int-factor.

$$\mu(t)v' - \mu(t)\frac{4}{t}v = -\frac{2}{t^2}\mu(t)$$

$$\mu'(t) = -\frac{4}{t}\mu(t)$$

$$\frac{\mu'}{\mu} = -\frac{4}{t}$$

$$\ln|\mu| = -4\ln|t| \qquad \mu = t^{-4}$$

$$t^{-4}v' - 4t^{-5}v = -2t^{2-4}$$

$$\cancel{(t^4v)' = -2t^3}$$

$$\cancel{t^4v = -\frac{1}{2}t^4 + C}$$

$$\cancel{v =}$$

$$(t^{-4}v)' = -2t^{-4}$$

$$t^{-4}v = \frac{2}{5}t^{-5} + C$$

$$v = \frac{2}{5t} + Ct^4$$

$$\frac{1}{y^2} = \frac{2}{5t} + Ct^4$$

$$y^2 = \frac{5t}{2 + 5Ct^5}$$

$$y = \pm\sqrt{\frac{5t}{2 + t^5 \cdot 5C}}$$

#30. $y' = \varepsilon y - \sigma y^3$

Let $v = y^{-2}$ $\quad y = v^{-1/2}$ $\quad y' = -\frac{1}{2}v^{-3/2}v'$

$$-\frac{1}{2}v^{-3/2}v' - \varepsilon v^{-1/2} = -\sigma v^{-3/2}$$

$$-\frac{1}{2}v' - \varepsilon v = -\sigma$$

$$v' + 2\varepsilon v = 2\sigma$$ Use $\mu = e^{2\varepsilon t}$

$$(e^{2\varepsilon t}v)' = 2\sigma e^{2\varepsilon t}$$

$$e^{2\varepsilon t}v = \frac{\sigma}{\varepsilon}e^{2\varepsilon t} + C$$

$$v = \frac{\sigma}{\varepsilon} + Ce^{-2\varepsilon t}$$

$$\frac{1}{y^2} = \frac{\sigma}{\varepsilon} + Ce^{-2\varepsilon t}$$

$$\frac{1}{y^2} = \frac{\sigma}{\varepsilon} + \frac{C\varepsilon e^{-2\varepsilon t}}{\varepsilon}$$

$$y^2 = \frac{\varepsilon}{\sigma + C\varepsilon e^{-2\varepsilon t}}$$

$$y = \pm\sqrt{\frac{\varepsilon}{\sigma + C\varepsilon e^{-2\varepsilon t}}}$$

11a. $y' = 5 - 3\sqrt{y}$ $\quad y(0) = 2$ $\quad h = .1$

$f_0 = 5 - 3\sqrt{2}$ $\qquad f(t,y) = 5 - 3\sqrt{y}$

$y_1 = 2 + .1(5 - 3\sqrt{2}) = 2.07574$

$f_1 = f(.1, 2.0757) = .6777$

$y_2 = y_1 + .1 \cdot f_1 = 2.14351$

$f_2 = f(.1, y_2) = .60778$

$y_3 = y_2 + .1 f_2 = 2.2043$

$f_3 = f(.1, y_3) = .5459$

$y_4 = y_3 + .1 f_3 = 2.2588$

$f_4 = f(.1, y_4) = .4911$

$y_5 = y_4 + .1 f_4 = \boxed{2.308}$ ← approx value at $t = .5 = 0 + 5h$

$\vdots$

$y_{10} = \boxed{2.4900}$

13. $\dfrac{4 - ty}{1 + y^2}$

i	t_i	y_i	f_i
1	.1	-1.92	.845
2	.2	-1.83	1.00
3	.3	-1.73	1.13
4	.4	-1.61	1.28
5	.5	$\boxed{-1.49}$	1.48
6	.6	-1.34	1.72
7	.7	-1.14	2.04
8	.8	-.964	2.47
9	.9	-.719	3.06
10	1.0	-.412	3.77

$\approx t = .5$

#15 .