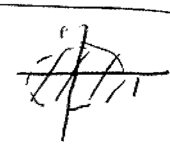
page 75 $\quad y' + \frac{y}{t(t-4)} = 0$ so $t \neq 0,4$ so $(0,4)$

2. ~~(t, ∞) since linear w/ $p(t)$ $g(t)$ continuous $\forall t$~~

8. $f(t,y) = \sqrt{1-t^2-y^2}$ is continuous inside $y^2+t^2 \le 1$

$\frac{df}{dy} = \frac{-y}{\sqrt{1-t^2-y^2}}$ " " " $y^2+t^2<1$ so

$$\boxed{y^2+t^2<1}$$



15. $y' + y^3 = 0 \quad y(0) = y_0$

$$\frac{dy}{dt} = -y^3 \qquad \frac{1}{y^3}\,dy = -dt$$

$$-\frac{1}{2}y^{-2} = -t + C$$

$$1 = 2y^2 t + C y^2$$

Now $\quad 1 = 0 + C y_0^2 \qquad C = 1/y_0^2$

$$1 = 2y^2 t + \frac{1}{y_0^2} y^2$$

$$1 = y^2 (2t + 1/y_0^2)$$

$$y = \sqrt{\frac{1}{2t + \frac{1}{y_0^2}}} = \frac{y_0}{\sqrt{2t y_0^2 + 1}}$$

Notice $y_0 = 0$ gives $y = 0$ equilibrium solution, valid $\forall t$.

If $y_0 \neq 0$ then $\quad 2t y_0^2 + 1 > 0$

so $\quad t > -\frac{1}{2y_0^2}$

22. $y' = \dfrac{-t + (t^2+4y)^{1/2}}{2}$ $y(2) = -1$

$y_1(t) = 1-t$ $y_2(t) = -t^2/4$ $y_1(2) = y_2(2) = -1$

$y_1' = -1 \stackrel{?}{=} \dfrac{-t+\sqrt{t^2+4-4t}}{2} = \dfrac{-t+\sqrt{(t-2)^2}}{2} = \dfrac{-t+|t-2|}{2}$

This is -1 only when when $t-2 \geq 0$

so solution holds $t \geq 2$

$y_2' = -\frac{t}{2} \stackrel{?}{=} \dfrac{-t+\sqrt{t^2-t^2}}{2} = -t/2$

This holds $\forall t$.

b Because $\frac{\partial f}{\partial y}$ is not continuous at the point $t=2$ $y=-1$

so the uniqueness part of the thm does not hold.

c. $y = ct + c^2$

$y' = c \stackrel{?}{=} \dfrac{-t+\sqrt{t^2+4ct+4c^2}}{2}$

$= \dfrac{-t+\sqrt{(t+2c)^2}}{2} = \dfrac{-t+|t+2c|}{2}$

This is $= c$ iff $t+2c \geq 0$

$t \geq -2c$

$c=-1$ gives $y = 1-t$

Clearly $ct+c^2$ is not $= -t^2/4$ for any c.

25. Given: $y_1' + p y_1 = 0$

$$\frac{y_2' + p y_2 = g(t)}{(y_1+y_2)' + p(y_1+y_2) = g(t)}$$

Thus $y_1 + y_2$ is a solution

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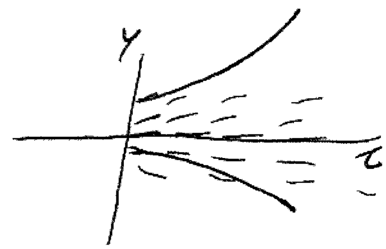
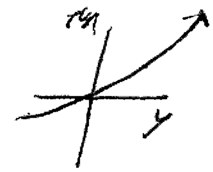
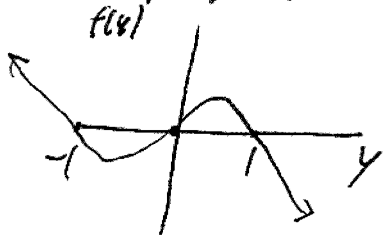
4. $y' = e^y - 1$ equil at $y=0$

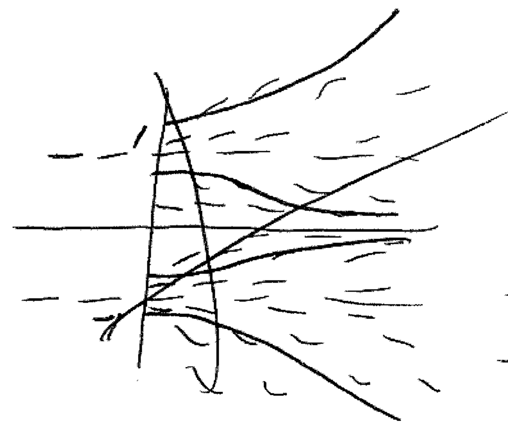


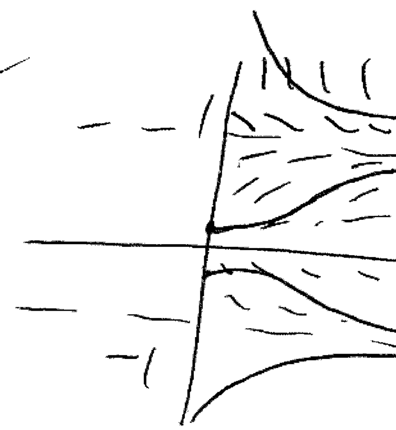


unstable equil at $y=0$.

10. $y' = y(1-y^2)$







$y = \pm 1$ are ~~unstable~~ stable
$y = 0$ is unstable

15. $y' = ry[1 - y/k]$ solution $y = \dfrac{y_0 K}{y_0 + (K - y_0)e^{-rt}}$ on page 82

a. ~~$2y_0 = \dfrac{y_0 K}{y_0 +}$~~ $y_0 = K/3$, set $y = 2K/3$ for doubling

$$\frac{2K}{3} = \frac{K^2/3}{K/3 + \frac{2K}{3}e^{-rt}}$$

$$2 = \frac{1}{1 + 2e^{-rt}}$$

$$2 + 4e^{-rt} = 1$$

$$e^{-rt} = -1/4$$

$$-rt = \ln(-\tfrac{1}{4})$$

$$T = \frac{-\ln(-\frac{1}{4})}{r}$$

$$t = \frac{\ln 4}{r}$$

$$\frac{\ln 4}{.025} = 55.5 \text{ yrs}$$

15 b. $y = \frac{y_0 K}{y_0 + (K - y_0) e^{-rt}}$ $\quad y_0 = K\alpha$

Solve for $y(T) = K\beta$

$$K\beta = \frac{K^2\alpha}{K\alpha + K(1-\alpha)e^{-rt}} \Rightarrow \beta = \frac{\alpha}{\alpha + (1-\alpha)e^{-rt}}$$

$$\beta\alpha + \beta(1-\alpha)e^{-rt} = \alpha$$

$$e^{-rt} = \frac{\alpha - \alpha\beta}{\beta(1-\alpha)} = \frac{\alpha(1-\beta)}{\beta(1-\alpha)}$$

$$-rt = \ln\left(\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\right)$$

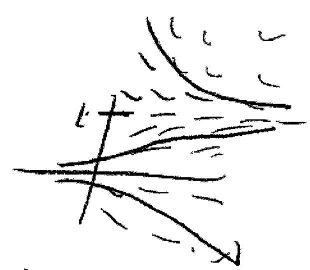
$$t = \frac{\ln\left(\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\right)}{-r}$$

Plug in $r = .025$, $\alpha = .1$ $\beta = .9$ gives $T = 176$ yrs

22. $x + y - 1$

$$\frac{dy}{dt} = \alpha y(1-y) \quad y(0) = y_0 \quad \alpha > 0$$

a. $y = 0$ & $y = 1$ are eq $y = 1$ is stable, $y = 0$ unstable



b. $\frac{1}{y(1-y)}\, dy = \alpha\, dt$

$\left(\frac{1}{y} + \frac{1}{1-y}\right) dy = \alpha\, dt$

$\ln|y| - \ln|1-y| = t + C$

$e^{\ln|y| - \ln|1-y|} = Ce^t$

$e^{\ln\left(\frac{y}{1-y}\right)} = Ce^t$

$\frac{y}{1-y} = Ce^t$

$y = Ce^t - Ce^t y$

$y = \cancel{Ce^t}$

22. $y = Ce^t - Ce^t y \rightarrow$

$y(0) = y_0$

$y_0 = C - Cy_0$

$C = y_0 / 1 - y_0$

$$y = \frac{Ce^t}{1 + Ce^t} = \frac{\frac{y_0}{1-y_0} e^t}{1 + \frac{y_0}{1-y_0} e^t} = \frac{y_0 e^t}{1 - y_0 + y_0 e^t}$$

which $\Rightarrow 1$
as $t \to \infty$

23. $\frac{dy}{dt} = -\beta y \qquad \frac{dx}{dt} = -\alpha x y$

a. $y(t) = y_0 e^{-\beta t}$

b. $\frac{dx}{dt} = -\alpha e^{-\beta t} x y_0$ this is separable

$\frac{1}{x} dx = -\alpha y_0 e^{-\beta t} dt$

$\ln|x| = y_0 \frac{\alpha}{\beta} e^{-\beta t} + C$ Given

$x = C e^{(\frac{\alpha}{\beta} e^{-\beta t} y_0)}$ given $x(0) = x_0$

$x_0 = C e^{\frac{\alpha}{\beta} y_0} \qquad C = x_0 e^{-\frac{\alpha}{\beta} y_0}$

$x(t) = x_0 e^{-\frac{\alpha}{\beta} y_0} e^{(\frac{\alpha}{\beta} e^{-\beta t} y_0)} = \boxed{x_0 e^{-\frac{\alpha y_0}{\beta}(1 - e^{-\beta t})}}$

c. As $t \to \infty$ $x(t) \to x_0 e^{\frac{-\alpha y_0}{\beta}}$