p265

2.

$y'' + \sin x\, y' + \cos x\, y = 0 \qquad y(0) = 0 \quad y'(0) = 1$

$y''(0) + 0 + 0 = 0 \qquad \boxed{y''(0) = 0}$

$y''' + \cos x\, y' + \sin x\, y'' - \sin x\, y' + \cos x\, y'' = 0$

$y'''(0) + 1 + 0 - 0 + 0 = 0 \qquad \boxed{y'''(0) = -1}$

$y^{(4)} + \cos x\, y'' - \sin x\, y' + \cos x\, y'' + \sin x\, y'''$

$- \sin x\, y'' - \cos x\, y' + \cos x\, y''' - \sin x\, y'' = 0$

$y^{(4)}(0) + 0 - 0 + 0 + 0 - 0 - 1 + 0 + 1 + 0 = 0 \qquad \boxed{y^{(4)}(0) = 2}$

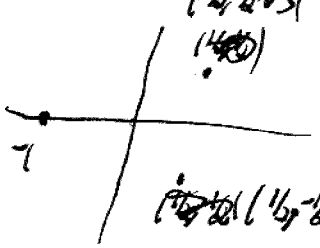
5 ∞ $\forall x_0$!

6 $y'' + \frac{x}{(x-3)(x+1)} y' + \frac{4}{(x-3)(x+1)} y = 0$

$x_0 = 4 \quad \rho = 1$
$x_0 = -4 \quad \rho = 3$
$x_0 = 0 \quad \rho = 1$

7 $y'' + \frac{4x}{1+x^3} y' + \frac{4}{1+x^3}$

roots are $x = -1, \ \frac{1}{2} + \frac{1}{2}\sqrt{3}\, i, \ \frac{1}{2} - \frac{1}{2}\sqrt{3}\, i$



$x_0 = 0$ distance is $\sqrt{\frac{1}{4} + \frac{3}{4}} = 1$ so $\rho = 1$

$x_0 = 2$ distance is $\sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$ $\rho = \sqrt{3}$

11.

$y'' + \sin x y = 0$

$y''' + \sin x y' + \cos x y = 0$

$y^{(4)} - \sin x y + 2\cos x y' + \sin x y''$

$y^{(5)} - \cos x y - 3\sin x y' + 3\cos x y'' + \sin x y'''$

$y^{(6)} + \sin x y - 4\cos x y' - 6\sin x y'' + 4\cos x y''' + \sin x y^{(4)}$

$y^{(7)} + \cos x y + 5\sin x y' - 11\cos x y'' - 10\sin x y''' + 5\cos x y^{(4)} + \sin x y^{(5)}$

Now let $y(0) = 0$ $y'(0) = 1$ and use equations:

$y''(0) = 0$ $y'''(0) = 0$ $y^{(4)}(0) = -2$

$y^{(5)}(0) = 0$ $y^{(6)}(0) = 4$ $y^{(7)}(0) = 10$

So $y_1(x) = x - \frac{2}{24}x^4 + \frac{4}{120}x^6 + \frac{10}{5040}x^7$

Also let $y(0) = 1$ $y'(0) = 0$

$y''(0) = 0$ $y'''(0) = -1$ $y^{(4)}(0) = 0$ $y^{(5)}(0) = 1$

$y^{(6)}(0) = 4$ $y^{(7)}(0) = 1$

$y_2(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{4}{720}x^6 + \frac{1}{5040}x^7$

Expect $\rho = \infty$ since $\sin x$ has $\rho = \infty$

p.278

1. $x^2y'' + 4xy' + 2y = 0$ $\alpha = 4$ $\beta = -2$

$r^2 + 3r + 2 = 0$ $r = -2, -1$

$y = c_1|x|^{-2} + c_2|x|^{-1}$

2. $(x+1)^2y'' + 3(x+1)y' + 0.75y$

~~$r^2 + 3r + \frac{3}{4}$~~ $(r+\frac{1}{2})(r+\frac{3}{2})$

Let ~~$t = x+1$~~

~~$t^2y'' + 3ty' + \frac{3}{4}y = 0$~~

$r^2 + 2r + \frac{3}{4} = (r+\frac{1}{2})(r+\frac{3}{2})$

$y = c_1|x+1|^{-\frac{1}{2}} + c_2|x+1|^{-\frac{3}{2}}$

4. $x^2y'' + 3xy' + 5y = 0$

$r^2 + 2r + 5 = 0$ $r = \frac{-2 \pm \sqrt{-16}}{2}$

$= \frac{-2 \pm 4i}{2} = -1 \pm 2i$

$y = |x|^{-1}[c_1\cos(2\ln|x|) + c_2\sin(2\ln|x|)]$

5. $x^2y'' - xy' + y = 0$ $r^2 - 2r + 1 = 0$ $(r-1)^2$

$y = (c_1 + c_2\ln|x|)|x|^r$