

## Exam 1 Solutions

1. a. A map  $\phi$  such that  $\phi(xy) = \phi(x)\phi(y) \quad \forall x, y \in G.$

b. A homomorphism which is 1-1 and onto

c. The least integer  $n > 0$  such that  $g^n = e.$

d. Permutations of  $\{1, 2, \dots, n\}$

e.  $\langle g \rangle = \{g^n \mid n \in \mathbb{Z}\}$

2. a. T b. F c. F d. T e. F f. F g. F h. T

3. a. yes b. no inverses c. yes d. not closed under +

e. No, 4 has no inverse f. yes g. yes h. no identity

4. a.  $geg^{-1} = e \quad \forall g$  so  $e$  is only conjugate to itself.

b. If  $G$  is abelian then  $g x g^{-1} = g g^{-1} x = x$  so  $x$  is only conjugate to itself.

c. Need  $g \sim g \quad \forall g \in G.$

$$g \sim h \Rightarrow h \sim g$$

$$g \sim h \text{ and } h \sim k \Rightarrow g \sim k$$

4 d.

i)  $X = eXe^{-1}$  so  $X \sim X$ .

ii) Suppose  $X \sim Y$  so  $gXg^{-1} = Y$  for some  $g$ . Then

$$Y = g^{-1}Xg = g^{-1}X(g^{-1})^{-1} \text{ so } Y \sim X.$$

iii) Suppose  $X \sim Y$  and  $Y \sim Z$ . Then  $\exists g_1, g_2 \in G$  so

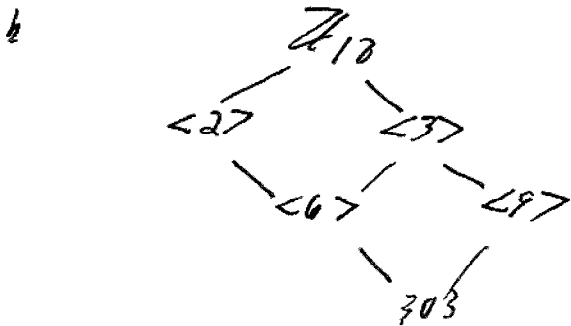
$$g_1Xg_1^{-1} = Y \text{ and } g_2Yg_2^{-1} = Z.$$

$$\text{Thus } Z = g_2Yg_2^{-1}$$

$$= g_2g_1Xg_1^{-1}g_2^{-1} = (g_2g_1)X(g_2g_1)^{-1}$$

$$\text{so } X \sim Z.$$

5. a.  $\{1, 5, 7, 11, 13, 17\}$



6. a. Let  $g_1, g_2 \in K$ . So  $\phi(g_1) = \phi(g_2) = e$ .

$\phi(g_1 g_2) = \phi(g_1) \phi(g_2) = ee = e$ . Thus  $g_1 g_2 \in K$ .

$\phi(g_1^{-1}) = \phi(g_1)^{-1} = e^{-1} = e$  so  $g_1^{-1} \in K$ .

Thus  $K \leq G$ .

b. The identity in  $\mathbb{R}^+$  is 1. Thus

$K = \{z \in \mathbb{C} \mid |z| = 1\}$  is the unit circle,  
which is a subgroup

Notice  $|z_1 z_2| = |z_1| |z_2|$  so  $\phi$  is a homom.

7. a. No

b.  $\sigma(1) = 2$   $\sigma(i) = \sigma(k) = 4$

c.  $-i$

d.  $(1, -1, k, -k)$

e. This group has only two elements which square to  $e$ .

$D_4$  has at least 4 (corresponding to the "flips")