1a. If $ghg^{-1} = gh'g^{-1}$ then $h = h'$ by cancellation.

Thus the # of ghg^{-1} is the # of elts in H.

So $|gHg^{-1}| = |H|$

b $e = geg^{-1} \in gHg^{-1}$

Let $ghg^{-1}, gh'g^{-1} \in gHg^{-1}$.

Then $(ghg^{-1})(gh'g^{-1}) = ghh'g^{-1} \in gHg^{-1}$

Also $(ghg^{-1})^{-1} = gh^{-1}g^{-1} \in gHg^{-1}$

Thus $gHg^{-1} \leq G$.

c $H = \{e, (12)(34), (12), (34)\}$

$g = (123)$

$gHg^{-1} = \{e, (23)(14), (23), (14)\} \leq S_4$

2 All subgroups of D_4 are listed on page 80.

The subgroups of order 4 are normal since they have index 2.

The only subgroups of size 2 which are normal is $\{e, r^2\}$

$\{e\}$ & D_4 are trivially normal

a. $N_G(H) = \{g \in G \mid gHg^{-1} = H\}$

Let $g_1, g_2 \in N_G(H)$, so $g_1 h g_1^{-1} \in H \ \forall h$

$g_2 h g_2^{-1} \in H \ \forall h$

$(g_1 g_2) h (g_1 g_2)^{-1} = g_1 g_2 h g_2^{-1} g_1^{-1}$ but $g_2 h g_2^{-1}$ is in H

$= g_1 h' g_1^{-1} \in H$ since $g_1 \in N_G(H)$

Thus $g_1 g_2 \in N_G(H)$

$g_1^{-1} h g_1$

Thus $g_1^{-1} \in N_G(H)$

So $N_G(H) \leq G$

b $H \trianglelefteq G$ iff $N_G(H) = G$

4 Let $A \in SL_n$ so $\det A = 1$

Let $X \in GL_n$

Then $\det(XAX^{-1}) = \det X \det A \det X^{-1}$

$= \det X \det A \dfrac{1}{\det X}$

$= \det A = 1$

Thus $XAX^{-1} \in SL_n$ so $SL_n \triangleleft GL_n$

4b $AH = BH$ iff
$A^{-1}B \in Sl_n(\mathbb{R})$
iff $\det(A^{-1}B) = 1$
iff $\det A = \det B$

a coset is all matrices w/ a given determinant

5. First notice that
$(gyg^{-1})^n = gy^ng^{-1}$.

Now $gag^{-1} = e$ if and only if $a = e$.

Thus $(gyg^{-1})^n = e$ if and only if $y^n = e$.

Thus order y = order gyg^{-1}

6. Let $g \in G$. By Lagrange, order $g = |\langle g \rangle| \,\big|\, |G|$.

Thus $|G| = \text{order}\, g \cdot k$

So $g^{|G|} = (g^{\text{order}\, g})^k = e^k = e$

7 Let $H \triangleleft G$, $K \triangleleft G$.

Let $x \in H \cap K$.

$gxg^{-1} \in H$ since $H \triangleleft G$

$gxg^{-1} \in K$ since $K \triangleleft G$

Thus $gxg^{-1} \in H \cap K$ so $H \cap K \triangleleft G$.

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2

Element	Order		Order		Order
(0,0)	1	(1,0)	3	(2,0)	3
(0,1)	4	(1,1)	12	(2,1)	12
(0,2)	2	(1,2)	6	(2,2)	6
(0,3)	4	(1,3)	12	(2,3)	12

yes it is cyclic $\cong \mathbb{Z}_{12}$

4. 15

7. 60

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1. yes 2. NO 3 yes 4 yes

2 points each

5. NO 6 yes 7 yes 8. NO

12 NO 13 yes 14. NO

29. ϕ_g is a homomorphism for all $g \in G$!

$\phi_g(xy) = gxyg^{-1}$

$= gxg^{-1}gyg^{-1} = \phi_g(x)\phi_g(y)$

49. Let $x,y \in G$.

$\nu\phi(xy) = \nu(\phi(xy))$

$= \nu(\phi(x)\phi(y))$ since ϕ is a homo.

$= \nu(\phi(x))\nu(\phi(y))$ since ν is a hom

$= \nu\phi(x)\,\nu\phi(y)$

so $\nu\phi$ is a homo.