SOLUTIONS

57.

36.

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

b $\langle 0 \rangle = \{0\}$ $\quad \langle 2 \rangle = \langle 4 \rangle = \{0, 2, 4\}$

$\langle 1 \rangle = \mathbb{Z}_6 = \langle 5 \rangle$

$\langle 3 \rangle = \{0, 3\}$

c 1, 5

d

$$\begin{tikzcd} & \mathbb{Z}_6 \arrow[dl, dash] \arrow[dr, dash] & \\ \langle 2 \rangle \arrow[dr, dash] & & \langle 3 \rangle \arrow[dl, dash] \\ & \{0\} & \end{tikzcd}$$

34. a. T b. F c. T d. F e. F
f. F g. F h. F i. T j. F

41. Let $x, y \in \phi(H)$ so $x = \phi(h_1)$, $y = \phi(h_2)$ for some $h_1, h_2 \in H$

Thus $xy = \phi(h_1)\phi(h_2) = \phi(h_1 h_2) \in \phi(H)$!

Also $x^{-1} = \phi(h_1)^{-1} = \phi(h_1^{-1}) \in \phi(H)$!

So $\phi(H)$ is closed under $\cdot$ and inverse

$\phi[H] \leq G'$

48. Let G be abelian.

Claim $H = \{x \in G \mid x^n = e\}$ is a subgroup

Pf.

Let $h_1, h_2 \in H$ so $h_1^n = h_2^n = e$. Then

$(h_1 h_2)^n = h_1^n h_2^n$ since G is abelian

$= e \cdot e = e$. Thus $h_1 h_2 \in H$.

Notice

$h_1 h_1^{n-1} = e$ so $h_1^{n-1} = h_1^{-1}$

$(h_1^{-1})^n = (h_1^{n-1})^n = (h_1^n)^{n-1} = e$

so $h_1^{-1} \in H$.

Thus $H \leq G$.

53. Let $H \leq G$. Define $a \sim b$ iff $ab^{-1} \in H$

a. $aa^{-1} = e \in H$ so $a \sim a$. ✓

b. Suppose $a \sim b$. Then $ab^{-1} \in H$. But $H \leq G$ so

$(ab^{-1})^{-1} \in H$

$\overset{\shortparallel}{ba^{-1}}$. Thus $b \sim a$ by def. ✓

c. Suppose $a \sim b$ and $b \sim c$. Then $ab^{-1} \in H$, $bc^{-1} \in H$.

But $H \leq G$ so

$ab^{-1}bc^{-1} \in H$

$\overset{\shortparallel}{ac^{-1}}$ so $a \sim c$ ✓

$\sim$ is an equiv. relation.

54. Done in class!

57. Done in class (we actually proved it must be cyclic <u>and</u> of prime order!

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10. 1, 5, 7, 11 4

11. 1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 51, 53, 57, 59 17

17. $\gcd(25, 30) = 5$ so $\langle 25\rangle = \langle 5\rangle = \{0, 5, 10, 15, 20, 25\}$ 6 elements

18. $\langle 30\rangle = \langle 6\rangle$ 7 elements

22.

$$\begin{tikzcd}
 & & \mathbb{Z}_{12} \arrow[dl, dash] \arrow[dr, dash] & \\
 & \langle 2\rangle \arrow[dl, dash] \arrow[dr, dash] & & \langle 3\rangle \arrow[dl, dash] \\
\langle 4\rangle \arrow[dr, dash] & & \langle 6\rangle \arrow[dl, dash] & \\
 & \{0\} & &
\end{tikzcd}$$

32. a. T b. F c. F d. F e. T
f. F g. F h. F i. T j. T

33. V the Klein 4 group is not cyclic   34. $\mathbb{Q}, +$

35. $\mathbb{Z}_2$ 36. None, only ∞ cyclic group is $\cong \mathbb{Z}$ and this has only 2 generators ± 1.

37. $\mathbb{Z}_5$ 1, 2, 3, 4 all generators

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1. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 6 & 5 & 4 \end{pmatrix}$ 2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 6 & 3 \end{pmatrix}$

3. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 5 & 6 & 2 \end{pmatrix}$ 4. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 6 & 2 & 4 & 3 \end{pmatrix}$

5. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 1 & 5 & 4 & 3 \end{pmatrix}$ 6. 6   7. 2

8. $\sigma^6 = i$ so $\sigma^{100} = \sigma^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 1 & 3 & 4 \end{pmatrix}$

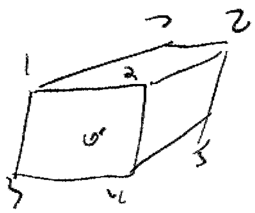
9. $\mu^2 = i$ so $\mu^{100} = i$.

11. $\{1, 3, 4, 5, 6, 2\}$ 12. $\{1, 2, 4, 3\}$ 13. $\{1, 5\}$

35. a. T b. F c. T d. T e. T
f. T g. F h. F i. F ($n=2$) j. T

44. First notice that D_n has $2n$ elements. For example there are n choices where to put the vertex labelled 1 and then 2 choices of orientation.

There are n rotations by $0, \frac{2\pi}{n}, \frac{4\pi}{n}, \dots \frac{2(n-1)\pi}{n}$ which form a subgroup w/ half the elements.

45. 

Notice 8 choices for where to send 1 then 3 choices for 2 then that "nails it down".

$|G| = 24$

- order 4 subgroups are rotations around each of 3 axes
- order 3 are rotations around long diagonals.