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2. Group

3. Not associative

$a*(b*c) = \sqrt{a\sqrt{bc}} \neq$

$(a*b)*c = \sqrt{\sqrt{ab}\,c}$

5. Not associative

6. No identity, notice $a*b$ is always in $\mathbb{R}$.

7. $G = \mathbb{Z}_{1000}$

8.

$\cdot_8$	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

19.

a. Since S is all real #'s except -1, to show it's a binary operation we just need to check $a*b$ is never -1

$$-1 = a+b+ab$$

$$a+b+ab+1 = 0$$

$$(a+1)(b+1) = 0 \leftarrow \text{never happens since } a \neq -1,\ b \neq -1$$

b. $(a*b)*c = a*b + c + (a*b)c$

$= a+b+ab+c + (a+b+ab)c$

$= a+b+ab+c + ac+bc+abc$

19b. $a*(b*c) = \cancel{a*b*} a + b*c + a(b*c)$

$= a + b + c + bc + a(b + c + bc)$

$= a + b + c + bc + ab + ac + abc$

so $a*(b*c) = (a*b)*c$ <u>associative</u>

Identity:

Need $a*e = a$

$a*e = a + e + ae$

$a = a + e + ae$

$e + ae = 0$ $\boxed{e = 0}$

<u>Inverse</u> set $a*x = 0$

$a + x + ax = 0$

$x + ax = -a$

$x(1+a) = -a$

$x = \dfrac{-a}{1+a}$

Thus $\boxed{a^{-1} = \dfrac{-a}{1+a}}$

$a * \dfrac{-a}{1+a} = 0$ which is the identity.

20.

G_1

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

G_2

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	a	e
c	c	b	e	a

G_3

	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

Since all elements in G_1 square to e, obviously G_1 is Not $\cong$ to G_2 or G_3

Claim Define $\varphi: G_2 \longrightarrow G_3$ by

$$e \longrightarrow e$$
$$a \longrightarrow b$$
$$b \longrightarrow c$$
$$c \longrightarrow a$$

Then φ is an $\cong$.

a. Yes both groups are commutative

b. G_2 is $\cong$ to U_4

c. check that $e \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $a \to \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $b \to \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $c \to \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is an $\cong$.

22. Axiom $\mathscr{L}_2$ must come before $\mathscr{L}_3$

25. a. F b. T c. T d. F e. F

f. T g. T h. T i. F j. T

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31 Suppose $x * x = x$. Then $x = e$ by left cancellation

36. Given: $(a*b)^{-1} = a^{-1} * b^{-1}$. But we know in any group that

$(a*b)^{-1} = b^{-1} * a^{-1}$. Thus

$a^{-1} * b^{-1} = b^{-1} * a^{-1}$ Now invert each side.

$(a^{-1} * b^{-1})^{-1} = (b^{-1} * a^{-1})^{-1}$

$b * a = a * b$ //

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1. yes   2. no (no inverses)   3. yes

4. yes   5. yes   6. No (not closed under addition)

7. 2 & 6.

21 a. 1, 2, 3, 4, 5

b. 1/2, 1/4, 1, 2, 4

c. $\pi, \pi^2, 1, \frac{1}{\pi}, \frac{1}{\pi^2}$

27. $3 +_4 3 = 2$ $3+3+3 = 1$ $3+3+3+3=0$

so order 4

28. 2, since $c * c = e$