29. NO 30. NO 31. yes. Equivalence classes all have 2 elements except $\{0\}$.

Classes: $\{0\}$, $\{a, -a\}$ for each $a > 0$.

32. NO 33. yes

$C_1 = \{1, 2, 3, \ldots, 9\}$
$C_2 = \{10, 11, 12, \ldots, 99\}$
$C_3 = \{100, 101, \ldots, 999\}$
$\vdots$

infinitely many equivalence classes

34. yes

$C_0 = \{10, 20, 30, 40, 50, \ldots\}$
$C_1 = \{1, 11, 21, 31, \ldots\}$
$C_2 = \{2, 12, 22, 32, \ldots\}$
$\vdots$
$C_9 = \{9, 19, 29, \ldots\}$

10 equivalence classes.

35. a. $\{0, 2, 4, 6, \ldots\}$
$\{1, 3, 5, 7, \ldots\}$

b. $\{0, 3, 6, 9, \ldots\}$
$\{1, 4, 7, 10, \ldots\}$
$\{2, 5, 8, 11, \ldots\}$

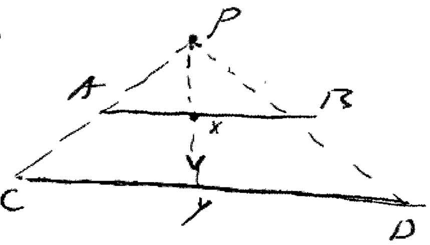
c. $\{0, 5, 10, 15, \ldots\}$
$\{1, 6, 11, 16, \ldots\}$
$\{2, 7, 12, 17, \ldots\}$
$\{3, 8, 13, 18, \ldots\}$
$\{4, 9, 14, 19, \ldots\}$

37. "one to one" functions are those that, for two distinct elements $x_1, x_2$ in the domain, $f(x_1)$ and $f(x_2)$ are two distinct elements in the range. Here "two-to-two" would be a better name.

p. 8

# Homework #1 Solutions

12. a. yes, not 1-1 or onto
    b. yes not 1-1 or onto
    c. No
    d. yes, 1-1 and onto
    e. yes, not 1-1 and onto
    f. No

13. 


This geometrically defines a bijection from points of $\overline{AB}$ to points of $\overline{CD}$.

16. a. $\emptyset$
    b. $\emptyset, \{a\}$
    c. $\emptyset, \{a\}, \{b\}, \{a,b\}$
    d. $\emptyset, \{a\}, \{b\}, \{c\}$
       $\{a,b\}, \{a,c\}, \{b,c\}$
       $\{a,b,c\}$

17. <u>Theorem:</u> Let $|A| = s$. Then $|P(A)| = 2^s$.

<u>Proof</u> We prove it by induction on $s$. It is obviously true for $s = 1, 2$ (see 16 b, c for instance!) So assume it is true for any set with $< s$ elements, and let $|A| = s$.

Fix a distinguished element $x \in A$. Notice that $P(A)$ is a disjoint union of subsets containing $x$ and those which don't, and

$$f: \begin{matrix}\text{subsets of } A\\ \text{w/out } x\end{matrix} \longrightarrow \begin{matrix}\text{subsets of } A\\ \text{with } x\end{matrix}$$

defined by $f(B) = B \cup \{x\}$ is obviously a bijection. Thus

$$|P(A)| = 2 \cdot \left|\begin{matrix}\#\text{ of subsets of}\\ A \text{ without } x\end{matrix}\right| = 2 \cdot |P(A - x)| = 2 \cdot 2^{s-1} \text{ by induction}$$

since $|A - x| = s - 1$. But $2 \cdot 2^{s-1} = 2^s$. //

p. 19

3. $-i$ 6. $26-2i$ 10. 5 14. $13\left(\frac{12}{13}+\frac{5}{13}i\right)$

18. $z^3=-8$ Notice $1, e^{\frac{2\pi i}{3}}$ and $e^{\frac{4\pi i}{3}}$ are solutions to $z^3=1$. Thus

$$-2,\ -2e^{\frac{2\pi i}{3}},\ -2e^{\frac{4\pi i}{3}}$$ are solutions.

20. $1, e^{\frac{2\pi i}{6}}, e^{\frac{4\pi i}{6}}, e^{\frac{6\pi i}{6}}, e^{\frac{8\pi i}{6}}, e^{\frac{10\pi i}{6}}$

23. 4 24. 14.8 38. See class notes!