

5.3 Improper Integrals (part 2)

Business application:

Recall the accumulated present value of a continuous income stream

R = yearly rate for c.i.s. (or $R(t)$ if it changes over time)

T = years for investment

k = interest rate

$$B = \int_0^T R e^{-kt} dt$$

Question: what would the present value be if one did not just invest for T years, but perpetually?

So instead of T years, ∞ years

$$B = \int_0^{\infty} R e^{-kt} dt$$

This is the accumulated present value of a c.i.s. for all time into the future.

This is also known as the capital value.

Example: A rental property nets \$1200/year paid into perpetuity.

Find the accumulated present value at 4% compounded continuously.

$$R = 1200 \quad k = .04$$

$$B = \int_0^{\infty} 1200 e^{-.04t} dt$$

$$= \lim_{b \rightarrow \infty} \int_0^b 1200 e^{-.04t} dt$$

$$= \lim_{b \rightarrow \infty} \left. \frac{1200}{-.04} e^{-.04t} \right|_0^b$$

$$= \lim_{b \rightarrow \infty} -30000 e^{-.04t} \Big|_0^b$$

$$= \lim_{b \rightarrow \infty} (-30000 e^{-.04b}) - (-30000 e^0)$$

$$= \quad \quad \quad 0 + 30,000$$

$$= \$30,000$$