

5.2 Applications of Integrating Growth and Decay Models

Final Exam
Monday Dec 14
Practice Final
Final Time Survey

Recall continuously compounded investments 3.3/3.4

Future Value
Present Value

$$A = P e^{kt}$$
$$P = A e^{-kt}$$

A continuous income stream is a recurring monetary income.
Example: Rent from a rental property.

If the money from this stream is invested, then the money invested at different times have different amounts of accrued interest invested at the end.

Accumulated Future Value of a Continuous Income Stream

$$A = \int_0^T R e^{k(T-t)} dt$$
$$= \boxed{e^{kT} \int_0^T R e^{-kt} dt}$$

where R = yearly rate for C.I.S.
 T = years for investment
 t = variable from 0 to T
 k = rate of investment

Recall: $\frac{d}{dx} e^{ax} = a \cdot e^{ax}$
 $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$

Ex There is a C.I.S. of \$1000 per year at 3%
 is invested over 10 years
 Find the A.F.V.

$$R = 1000 \quad T = 10 \quad k = 0.03$$

$$\text{A.F.V.} = e^{(0.03)(10)} \int_0^{10} 1000 e^{-(0.03)t} dt$$

$$= e^{0.3} \cdot \left(\frac{1000}{-0.03} e^{-0.03t} \right) \Big|_0^{10}$$

$$= e^{0.3} \left(\frac{1000}{-0.03} e^{-0.3} - \frac{1000}{-0.03} e^0 \right)$$

$$= e^{0.3} (-24,693.94 + 33,333.33)$$

$$= e^{0.3} \cdot 8639.39$$

$$= 1.3498588 \cdot 8639.39$$

$$= \$11,661.96$$

The Accumulated Present Value of a C.I.S.
 = present value of the A.F.V. of a C.I.S.

$$B = \int_0^T R e^{-kt} dt$$

Ex Find the A.P.V. of a C.I.S.
of \$500 per year with an interest
rate of 5% over 3 years.

$$\begin{aligned} \text{A.P.V.} &= \int_0^3 500 e^{-0.05t} dt \\ &= \frac{500}{-0.05} e^{-0.05t} \Big|_0^3 \\ &= \frac{500}{-0.05} e^{-0.15} - \frac{500}{-0.05} e^0 \\ &= -8,607.08 + 10,000 \\ &= \$1,392.92 \end{aligned}$$

For each question

- One time investment or C.I.S.
- Future Value or Present Value