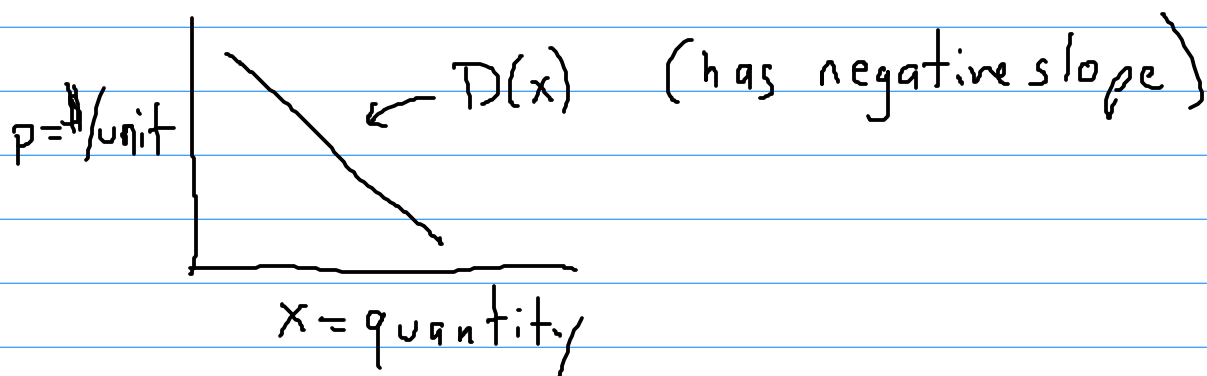
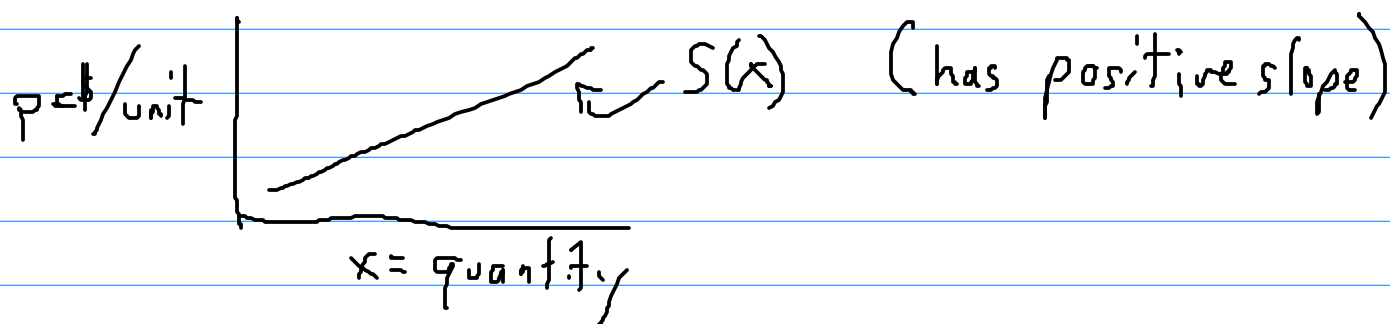


5. | An Economics Application: Consumer Surplus and Producer Surplus

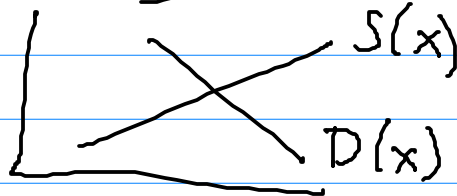
Demand $p = D(x)$ = price/unit consumer is willing to pay for x units



Supply $p = S(x)$ = price/unit producer is willing to accept for selling x units.



Modeling Supply and Demand Together



Where the supply and demand curves intersect is called equilibrium point.

Q = equilibrium quantity

P = equilibrium price

$\left\{ \begin{array}{l} = x_E \\ = p_E \end{array} \right.$

Ex Find the equilibrium quantity and equilibrium price for

$$S(x) = 2x \quad D(x) = 30 - x$$

$$S(x) = D(x)$$

$$2x = 30 - x \rightarrow 3x = 30$$

$$x = 10$$

$$\begin{array}{l} S(10) \\ 2(10) \\ 20 \end{array}$$

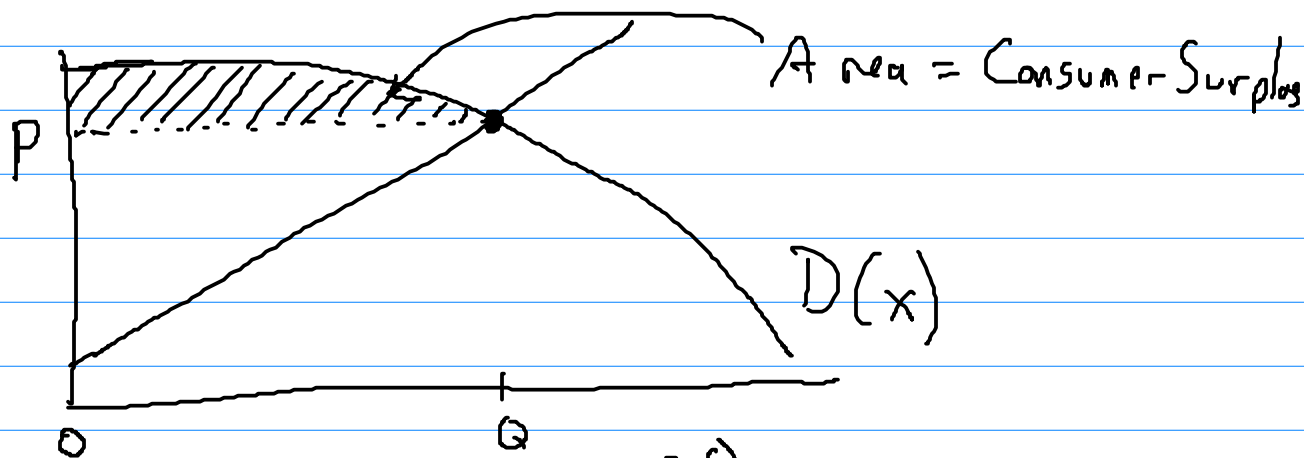
$$\begin{array}{l} D(10) \\ 30 - 10 \\ 20 \end{array}$$

$$Q = 10$$

$$P = 20$$

point
(10, 20)

Consumer Surplus = Difference between what a consumer is willing to pay and the market price.



$$\text{Consumer Surplus} = \int_0^Q D(x) - P \, dx$$

$$= \int_0^Q D(x) \, dx - P \cdot Q$$

Find the Consumer Surplus for our example

$$D(x) = 30 - x \quad Q = 10 \quad P = 20$$

$$C.S. = \int_0^{10} 30 - x \, dx - 20 \cdot 10$$

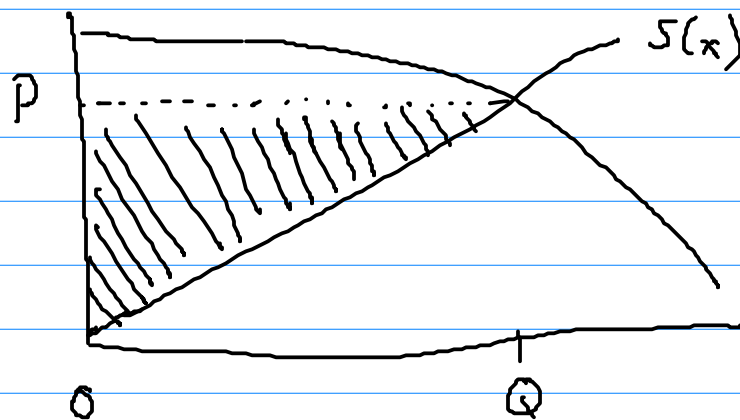
$$= 30x - \frac{1}{2}x^2 \Big|_0^{10} - 200$$

$$= \left[\overset{F(10)}{300} - \overset{F(0)}{50} \right] - [0 - 0] - 200$$

$$= 250 - 200$$

$$= \$50$$

Producer Surplus = difference between the market price and what a producer is willing to accept



$$\text{Producer Surplus} = \int_0^Q P - S(x) \, dx$$

$$= P \cdot Q - \int_0^Q S(x) \, dx$$

Find the Producer Surplus in our example

$$S(x) = 2x \quad Q = 10 \quad P = 20$$

$$\begin{aligned} \text{P.S.} &= 20 \cdot 10 - \int_0^{10} 2x \, dx \\ &= 200 - x^2 \Big|_0^{10} \\ &= 200 - \left(\frac{F(10)}{100} - \frac{F(0)}{0} \right) \\ &= 200 - 100 \\ &= \$100 \end{aligned}$$

Ex

$$\begin{aligned} S(x) &= 3x^2 - 4x + 200 \\ D(x) &= 500 - 4x \end{aligned}$$

a) Find the equilibrium point

$$\begin{aligned} S(x) &= D(x) \\ 3x^2 - 4x + 200 &= 500 - 4x \\ 3x^2 &= 300 \end{aligned}$$

$$x^2 = 100$$

$$x = 10, -10$$

$$D(10) = 500 - 4(10) = 460$$

$$Q = 10$$

$$P = 460$$

b) Find the consumer surplus

$$C.S. = \int_0^{10} 500 - 4x \, dx - (460)(10)$$

$$= 500x - 2x^2 \Big|_0^{10} - 4600$$

$$= \left[\overset{F(10)}{5000} - 200 \right] - \left[\overset{F(0)}{0 - 0} \right] - 4600$$

$$= 4800 - 0 - 4600$$

$$= \$200$$