4.5 Methods of Integration: Substitution

More Examples

Method:

1. Pick u as the "inside function"
2. Find the derivative $\frac{du}{dx}$ Solve for dx=
3. Replace all x and dx with u and du
4. Integrate in terms of u
5. Change all instances of u back into x

Step 1: $(\underset{u}{\square})^{\#}$ $e^{\underset{u}{\square}}$ $\ln(\underset{u}{\square})$ $\sqrt{\underset{u}{\square}}$

Ex $\int (\ln x)^3 \frac{1}{x}\, dx$

① $u = \ln x$

② $\frac{du}{dx} = \frac{1}{x} \rightsquigarrow x\,du = 1\cdot dx$ $\quad dx = x\,du$

③ $= \int (u)^3 \cdot \frac{1}{x} \cdot x \cdot du$

$= \int u^3\, du$

④ $= \frac{1}{4}u^4 + C$

⑤ $= \frac{1}{4}(\ln x)^4 + C$

Ex $\int \frac{dx}{3x+5}$ or $\int \frac{1}{3x+5}\,dx$

① $u = 3x+5$

② $\frac{du}{dx} = 3 \rightsquigarrow du = 3\,dx \rightsquigarrow dx = \frac{du}{3}$

③ $= \int \frac{1}{u} \cdot \frac{du}{3}$

$= \int \frac{1}{3} \cdot \frac{1}{u}\,du$

④ $= \frac{1}{3} \ln u + C$

⑤ $= \frac{1}{3} \ln(3x+5) + C$

w/ endpoints

Idea: $\int_5^7 \sim\sim dx$

$u = 3x$ $\quad u(5) = 15$, $u(7) = 21$

$\rightarrow \int_{15}^{21} \sim\sim du$

Ex $\int_{-1}^{1} e^{x^2} \cdot 2x\, dx$

① $u = x^2$

② $\frac{du}{dx} = 2x \rightsquigarrow du = 2x\,dx \rightsquigarrow dx = \frac{du}{2x}$

E $u(1) = (1)^2 = 1$

$u(-1) = (-1)^2 = 1$

③ $\int_1^1 e^u \cancel{2x} \frac{du}{\cancel{2x}}$

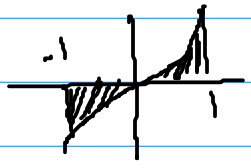
$= \int_1^1 e^u du$

④ $= e^u \Big|_1^1$

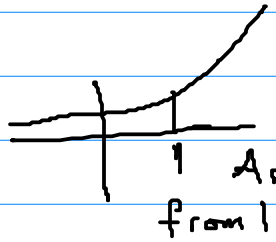
$F(b) - F(a)$

$= [e^1] - [e^1]$

$= 0$



Area in x $= 0$



Area in u from 1 to 1 $= 0$