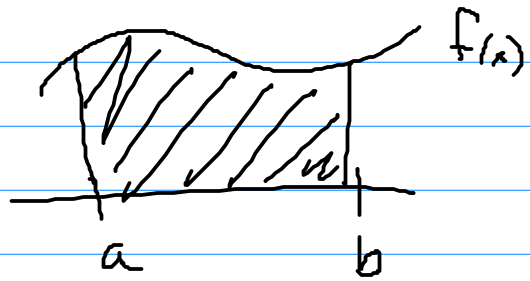
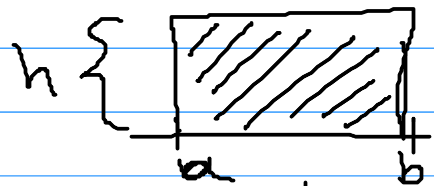
4.4 Properties of Definite Integrals (part 2)

Background

The (signed) area under the curve y=f(x) between x=a and x=b is $\int_a^b f(x)\,dx$



Consider a rectangle over [a,b] of height h



the width = $b-a$

the area = $h(b-a)$

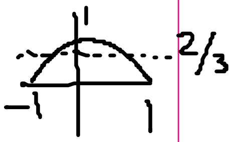
What height should the rectangle be so that the area of the rectangle equals the area under the curve?

$$\frac{h(b-a)}{b-a} = \frac{\int_a^b f(x)\,dx}{b-a}$$

$$h = \frac{1}{b-a}\int_a^b f(x)\,dx$$

The <u>average</u> value of f(x) between x=a and x=b is

$$\boxed{y_{av} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$



$a=-1 \quad b=1$

Ex Find the average value of $f(x)=1-x^2$ from $x=-1$ to $x=1$

$$y_{av} = \frac{1}{1-(-1)} \int_{-1}^{1} 1-x^2\,dx$$
$$= \frac{1}{2} \int_{-1}^{1} 1-x^2\,dx$$
$$= \frac{1}{2}\left(x-\frac{1}{3}x^3\right)\Big|_{-1}^{1}$$
$$= \left[\frac{1}{2}\left(1-\frac{1}{3}\right)\right] - \left[\frac{1}{2}\left(-1+\frac{1}{3}\right)\right]$$
$$= \left[\frac{1}{3}\right] - \left[-\frac{1}{3}\right]$$
$$= \frac{1}{3} + \frac{1}{3} = 2/3$$

Ex Find the average value of $f(x)=3x+7x^3$ from $x=0$ to $x=4$

$$y_{av} = \frac{1}{4-0} \int_{0}^{4} 3x+7x^3\,dx$$
$$= \frac{1}{4}\left(\frac{3}{2}x^2+\frac{7}{4}x^4\right)\Big|_{0}^{4}$$
$$= \frac{3}{8}x^2+\frac{7}{16}x^4\Big|_{0}^{4}$$
$$= \left[\frac{3}{8}\cdot 16+\frac{7}{16}\cdot 256\right] - \left[\frac{3}{8}\cdot 0+\frac{7}{16}\cdot 0\right]$$
$$= 3\cdot 2 + 7\cdot 16 - 0$$
$$= 6 + 112$$
$$= 118$$

Ex The weekly sales of a given product was shown to be $S(t) = 8e^{t}$ in dollars, t weeks after launch. Find the average sales between weeks 2 and 6 (t=2 and t=6).

$$\text{avg sales} = \frac{1}{6-2}\int_2^6 8e^t\,dt$$

$$= \frac{1}{4}\cdot 8e^t \Big|_2^6$$

$$= 2e^t\Big|_2^6$$

$$= 2e^6 - 2e^2$$

$$= 2\cdot 403.4288 - 2\cdot 7.3890$$

$$= 806.8576 - 14.7780$$

$$= 792.0796$$

$$= \$792.08$$