Test 3
Nov 23

4.3 Area and the Definite Integral

Antiderivative Review:

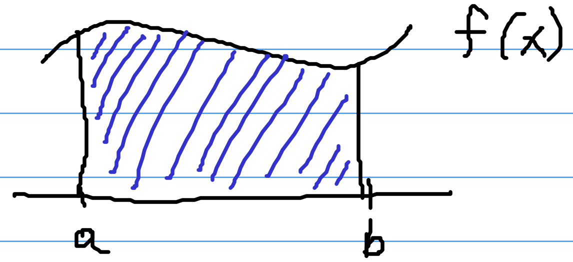
$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

$$\int x^3 dx = \frac{1}{4} x^4 + C$$

$$\int 8x\, dx = 8 \cdot \frac{1}{2} x^2 + C = 4x^2 + C$$

$$\int \frac{1}{x^3} dx = \int x^{-3} dx = -\frac{1}{2} x^{-2} + C$$

Recall: $\int_a^b f(x)dx$ = area under $f(x)$ from a to b



If $F(x)$ is any antiderivative of $f(x)$ then

$$\boxed{\int_a^b f(x)dx = F(b) - F(a)}$$

Shortcut Notation

$$F(x)\Big|_a^b = F(b) - F(a)$$

so $\int_a^b f(x)dx = F(x)\Big|_a^b$ ① antider

$= F(b) - F(a)$ ②

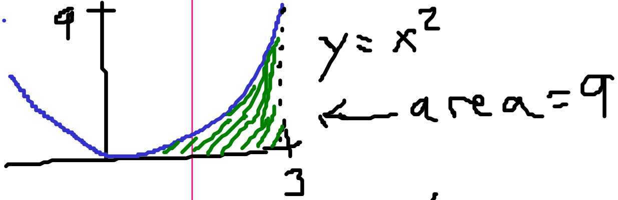
Ex $\int x^2 dx = \frac{1}{3}x^3 + C$ general antider

Ex $\int_0^3 x^2 dx = \frac{1}{3}x^3 + C\Big|_0^3$ area

$= [9 + C] - [0 + C]$

$= 9 + C - C$

$= 9$



The C's cancel out so do not include them in definate integrals.

Ex $\int_0^2 6x + 7\,dx = 3x^2 + 7x \Big|_0^2$ (no +C)

$$= \overset{F(2)}{[12 + 14]} \overset{-}{-} \overset{F(0)}{[0 + 0]}$$

$$= 26$$

Ex $\int_0^1 x^3 + x^{1/3}\,dx = \frac{1}{4}x^4 + \frac{3}{4}x^{4/3} \Big|_0^1$

$$= \overset{F(1)}{\left[\frac{1}{4} + \frac{3}{4}\right]} \overset{-}{-} \overset{F(0)}{[0 + 0]}$$

$$= 1$$

Ex Find exactly $\int_1^{13} \frac{1}{x}\,dx$

$$= \ln x \Big|_1^{13}$$

$$= \ln 13 - \ln 1$$

$\left(\ln\left(\frac{x}{y}\right) = \ln x - \ln y \right.$ or $\left. \ln 1 = 0 \right)$ $\quad = \ln \frac{13}{1} = \ln 13$

≈ 2.56949357

Ex Find exactly $\int_2^5 e^x\,dx$

$$= e^x \Big|_2^5$$

$$= e^5 - e^2$$

≈ 141.024103

$\Big\{$ or MML $e^2(e^3 - 1)$

What is the area under $y = 2x$ from 0 to 5?

Ex $\int_0^5 2x\,dx = x^2\Big|_0^5 = 25 - 0 = 25$

Ex $\int_2^3 x^2 - x + 1\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\Big|_2^3$

$F(3) - F(2)$

$= \left[9 - \frac{9}{2} + 3\right] - \left[\frac{8}{3} - 2 + 2\right]$

Common Denominator 6

$= \left[\frac{54}{6} - \frac{27}{6} + \frac{18}{6}\right] - \left[\frac{16}{6} - \frac{12}{6} + \frac{12}{6}\right]$

$= \frac{45}{6} - \frac{16}{6}$

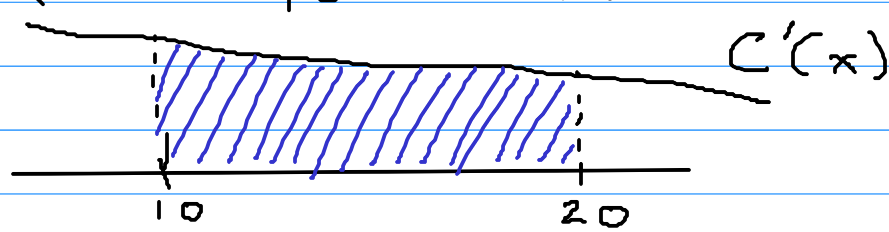
$= \frac{29}{6}$

Ex The Marginal Cost for producing x items is $C'(x) = 700 - 4x$

Find the total cost for producing 10 additional units when 10 are already made.

Recall: Total Cost = Area under Marginal Cost

start # = 10
end # = 10 + 10 = 20



$$\text{Area} = \int_{10}^{20} 700 - 4x \, dx$$

$$= 700x - 2x^2 \Big|_{10}^{20}$$

$$\qquad F(20) \qquad\qquad - \qquad\qquad F(10)$$

$$= [14000 - 800] - [7000 - 200]$$

$$= 13200 - 6800$$

$$= \$ \, 6,400$$