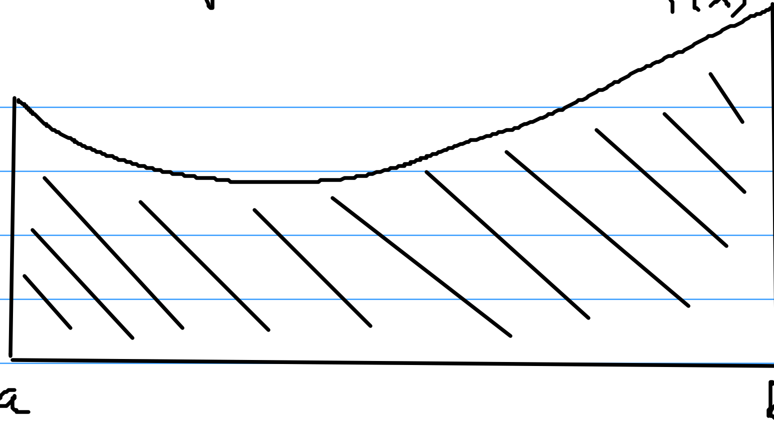
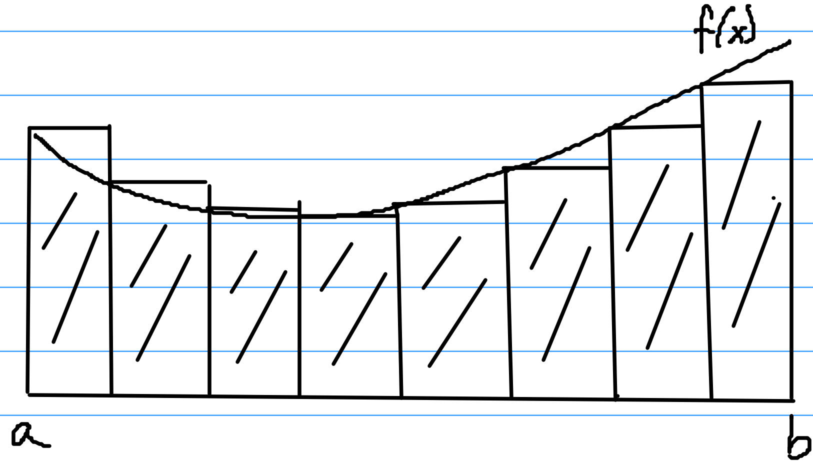
4.2 Example



Definition $\int_a^b f(x)\,dx$ = area under f(x) from a to b



Approximation of the area using n equal width rectangles

- Each width is the same $\Delta x = \frac{b-a}{n}$
- For the i^{th} rectangle:
  - x_i is the left endpoint
  - $x_1 = a \qquad x_{i+1} = x_i + \Delta x$
- The height of the i^{th} rectangle is $f(x_i)$
- The area of the i^{th} rectangle is $f(x_i) \cdot \Delta x$
- Area of all n rectangles

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

- Actual area

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Ex: ~~Find~~ Approximate the area under $f(x) = x^2 + 2x + 1$
on the interval [-1, 7] using n = 4 rectangles

$$\text{width} = b - a = 7 - (-1) = 8$$
$$\Delta x = \frac{b-a}{n} = \frac{8}{4} = 2$$

i	x_i	$f(x_i)$	$f(x_i)\cdot\Delta x$
1	-1	0	0
2	1	4	8
3	3	16	32
4	5	36	72

Scratch

$x_1 = a = -1$
$x_2 = x_1 + \Delta x = -1 + 2$
etc $+\Delta x$, $+\Delta x$

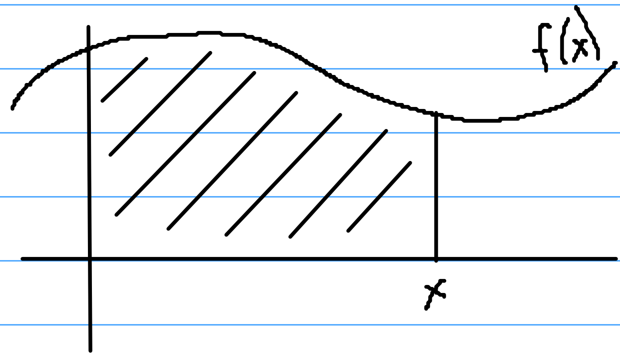
$f(-1) = 1 - 2 + 1 = 0$ $\quad 0 \cdot 2 = 0$
$f(1) = 1 + 2 + 1 = 4$ $\quad 4 \cdot 2 = 8$
$f(3) = 9 + 6 + 1 = 16$ $\quad 16 \cdot 2 = 32$
$f(5) = 25 + 10 + 1 = 36$ $\quad 36 \cdot 2 = 72$

Approximate area =

$$\sum_{i=1}^{4} f(x_i)\Delta x = 0 + 8 + 32 + 72$$
$$= 112$$

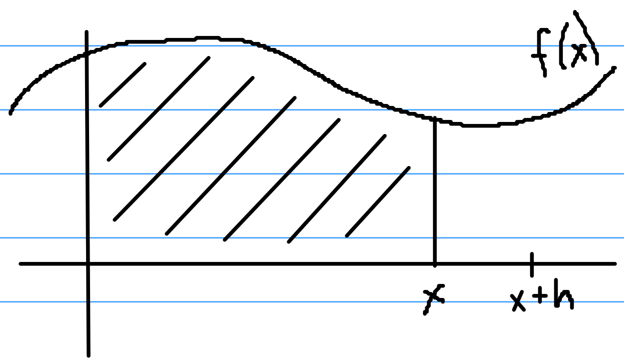
4.3 Area and Definate Integrals (Background)

Let $f(x)$ be a non negative function.
Let $A(x)$ = area under $f(x)$ from $a=0$ to $b=x$

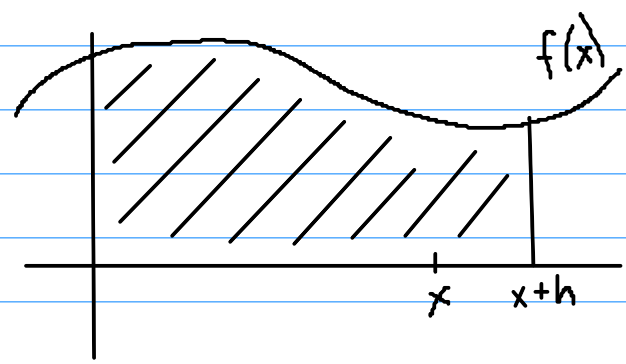


Analyze $A'(x)$

Recall $$A'(x) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h}$$

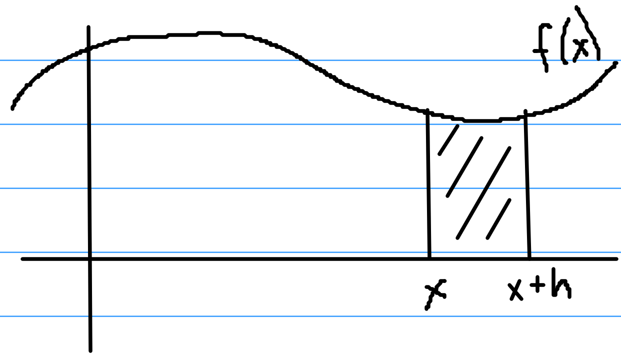


A(x)



A(x+h)

$$A(x+h) - A(x) = ?$$



$$A(x+h) - A(x)$$

When h is small, this tiny region has an area close to that of a tiny rectangle with height f(x) and width h.

So $A(x+h) - A(x) \approx f(x) \cdot h$

Thus $$A'(x) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x) \cdot h}{h}$$

$$= \lim_{h \to 0} f(x)$$

$$= f(x)$$

Thus the derivative of the area A(x) is the function f(x).

Thus the area under f(x) is an antiderivative of f(x).