4.1 Antidifferentiation (part 2)

$\int f(x)\,dx = F(x) + C$

where $F'(x) = f(x)$

C is a generic constant

$\int f(x)\,dx$ is the generic antiderivative of $f(x)$

$$\boxed{\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C} \qquad n \neq -1$$

$$\boxed{\int e^x\,dx = e^x + C}$$

Recall Chain Rule $\qquad \frac{d}{dx}e^{ax} = a e^{ax}$

$$\boxed{\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C}$$

$$\boxed{\int \frac{1}{x}\,dx = \ln(x) + C}$$

$= \int x^{-1}\,dx$

Ex $\int e^{x}+e^{3x}\,dx$

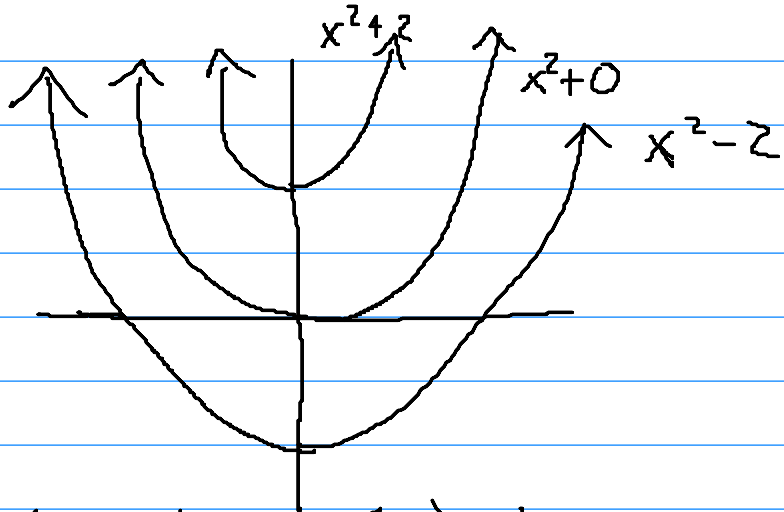
$$= e^{x}+\frac{1}{3}e^{3x}+C$$

Ex $\int x+1+x^{-1}+x^{-2}\,dx$

$$= \frac{1}{2}x^{2}+x+\ln x+\frac{1}{-1}x^{-1}+C$$

(or $-x^{-1}$)

The antiderivatives of $2x$ are all of the form $x^{2}+C$



which antiderivative $F(x)$ has $F(2)=3$
i.e. goes through the point $(2,3)$

$$F(x)=x^{2}+C$$

$$3=F(2)=4+C$$

$$C=-1$$

$$F(x)=x^{2}-1$$

4.2 Areas

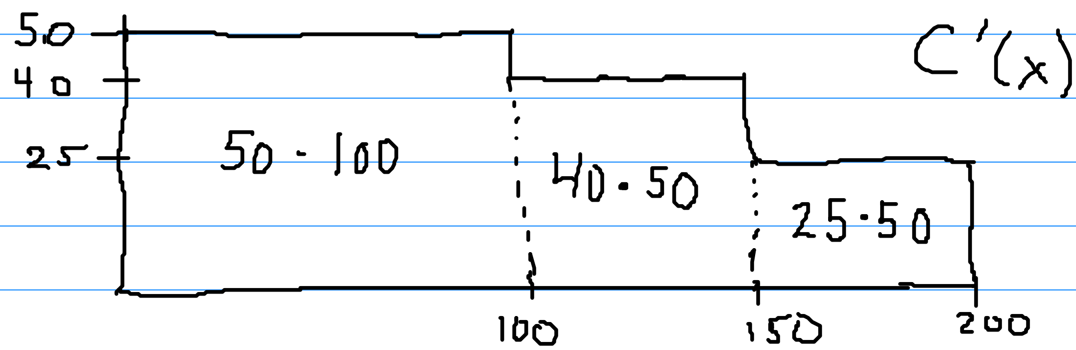
Recall: Marginal Cost C'(x) is the cost to produce one more unit with x already produced.

Suppose

$$C'(x) = \begin{cases} \$50/\text{item} & 0 \le x < 100 \\ \$40/\text{item} & 100 \le x < 150 \\ \$25/\text{item} & 150 \le x < 200 \end{cases}$$

Find the total cost for producing 200 items.

Find $C(200)$.

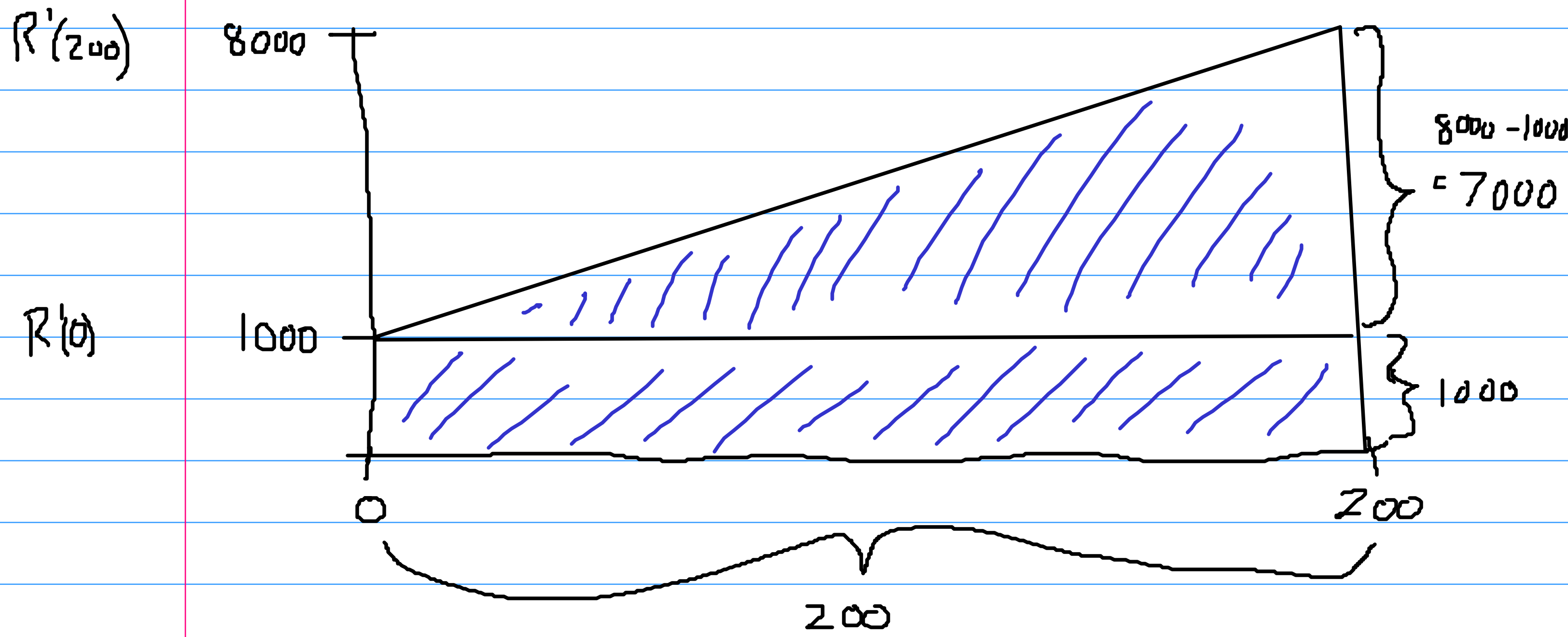


$$\begin{aligned} \text{Total Cost} &= 50 \cdot 100 + 40 \cdot 50 + 25 \cdot 50 \\ &= 5{,}000 + 2{,}000 + 1{,}250 \\ &= \$8{,}250 \end{aligned}$$

The area under the marginal cost/revenue/profit curve is the total cost/revenue/profit.

Ex Marginal Revenue $R' = 1000 + 35x$

Use geometry to find the total revenue for the first 200 items sold.



Total Revenue = Total Area

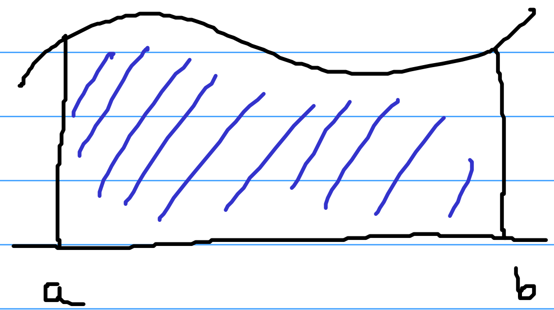
= Area of Rectangle + Area of Triangle

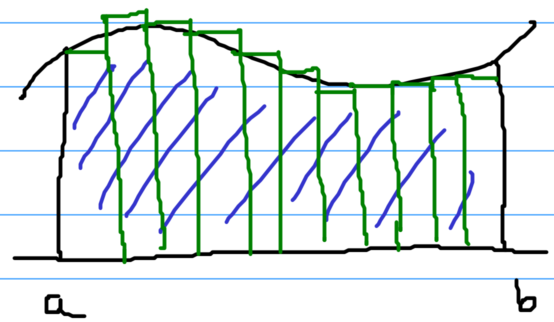
$= 200 \cdot 1000 + \frac{1}{2} \cdot 200 \cdot 7000$

$= 200{,}000 + 700{,}000$

$= \$900{,}000$

Approximating the area under a graph using n equal width rectangles.



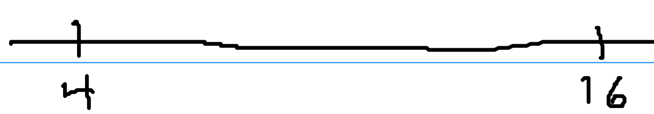


a = start x value
b = end x value
width = b − a
n = # of rectangles

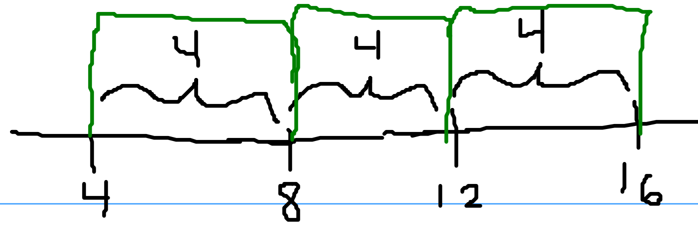
Δx = width of a rectangle $= \dfrac{\text{width}}{n} = \dfrac{b-a}{n}$

Example (setup only)

$a=4 \quad b=16 \quad n=3$ rectangles



$$\Delta x = \frac{16-4}{3} = \frac{12}{3} = 4$$



Summation Notation

If $a_1, a_2, \ldots, a_n$ are real numbers

then $\displaystyle\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$

i is the index of summation
1 is the start
n is the end

Ex $\displaystyle\sum_{i=1}^{3} e^i = e^1 + e^2 + e^3$

Ex $\displaystyle\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2$

Def for Exact Area

$\displaystyle\int_a^b f(x)\,dx$ = exact area under $f(x)$ from a to b