3.5 The Derivatives of a^x and $\log_a x$

Recall $\frac{d}{dx} e^x = e^x$

New $\boxed{\frac{d}{dx} a^x = \ln a \cdot a^x}$

Ex $y = 4^x$
$y' = \ln 4 \cdot 4^x$

Ex $y = 3.2^x$
$y' = \ln 3.2 \cdot 3.2^x$

Recall $\frac{d}{dx} e^{g(x)} = e^{g(x)} \cdot g'(x)$ Chain Rule

New $\boxed{\frac{d}{dx} a^{g(x)} = \ln a \cdot a^{g(x)} \cdot g'(x)}$

Ex $y = 7^{5x^2-2x}$
$y' = \ln 7 \cdot 7^{5x^2-2x} \cdot (10x-2)$

Recall $\frac{d}{dx} \ln x = \frac{1}{x}$ (Friday Video #1)

New $\boxed{\frac{d}{dx} \log_a x = \frac{1}{\ln a} \cdot \frac{1}{x}}$

Ex $y = \log_5 x$
$y' = \frac{1}{\ln 5} \cdot \frac{1}{x}$

Ex $y = 4 \cdot \log x = 4 \cdot \log_{10} x$
$y' = 4 \cdot \frac{1}{\ln 10} \cdot \frac{1}{x} = \frac{4}{\ln 10} \cdot \frac{1}{x}$

Recall $\frac{d}{dx} \ln(g(x)) = \frac{g'(x)}{g(x)}$ OR $\frac{1}{g(x)} \cdot g'(x)$

Chain Rule

New $\boxed{\frac{d}{dx} \log_a (g(x)) = \frac{1}{\ln a} \cdot \frac{g'(x)}{g(x)} \text{ OR } \frac{1}{\ln a} \cdot \frac{1}{g(x)} \cdot g'(x)}$

Ex $y = \log_2 (x^2+3x+1)$

$y' = \frac{1}{\ln 2} \cdot \frac{2x+3}{x^2+3x+1}$ OR $\frac{1}{\ln 2} \cdot \frac{1}{x^2+3x+1} \cdot (2x+3)$

Quiz $y = \log_3 (7-18x^2)$

$y' = \frac{1}{\ln 3} \cdot \frac{-36x}{7-18x^2}$ or $\frac{1}{\ln 3} \cdot \frac{1}{7-18x^2} (-36x)$

4.1 Antidifferention

Def An antiderivative of a function $f(x)$ is a function $F(x)$ where $F'(x) = f(x)$.

Example an antiderivative of $f(x) = 2x - 3$
is $F(x) = x^2 - 3x - 8$

Check $F'(x) = f(x)$ ✓

$$x^2-3x-8 \underset{\text{anti}}{\overset{\text{der}}{\rightleftarrows}} 2x-3 \underset{\text{anti}}{\overset{\text{der}}{\rightleftarrows}} 2$$

But there is an issue.
Another antiderivative of $f(x) = 2x-3$
is $F(x) = x^2 - 3x + 5$

They differ by a constant.
And $\frac{d}{dx}(\text{constant}) = 0$

The constant can be anything.

Def The general antiderivative of $f(x)$ is $F(x) + C$
where $F'(x) = f$ and C is a generic constant

So the general antiderivative of $f(x) = 2x - 3$
is $F(x) = x^2 - 3x + C$

Notation: The general antiderivative of $f(x)$ is also written

$$\int f(x)\, dx$$

This is the (indefinite) integral of $f(x)$

$$\int f(x)\, dx = F(x) + C$$

$$\boxed{\int x^n\, dx = \frac{1}{n+1} x^{n+1} + C} \quad n \neq -1$$

Ex $\int x\, dx = \frac{1}{2} x^2 + C$

Ex $\int x^3\, dx = \frac{1}{4} x^4 + C$

Ex $\int \frac{1}{x^3}\, dx = \int x^{-3}\, dx = \frac{1}{-2} x^{-2} + C$

Ex $\int \sqrt{x}\, dx = \int x^{1/2}\, dx = \frac{2}{3} x^{3/2} + C$

reciprical

Ex $\int 1\, dx = x + C$

$$\boxed{\int e^x\, dx = e^x + C}$$

$$\boxed{\int e^{ax}\, dx = \frac{1}{a} e^{ax} + C}$$

$$\boxed{\int \frac{1}{x}\, dx = \int x^{-1}\, dx = \ln x + C}$$

Ex $\int 6x^5 + 6x^2 + 6x + 7 \, dx$

$= 6 \cdot \frac{1}{6}x^6 + 6 \cdot \frac{1}{3}x^3 + 6 \cdot \frac{1}{2}x^2 + 7x + C$

$= x^6 + 2x^3 + 3x^2 + 7x + C$

Ex $\int 15x^{1/2} + 15x^{-4} \, dx$

$= 15 \cdot \frac{2}{3}x^{3/2} + 15\left(\frac{-1}{3}\right)x^{-3} + C$

$= 10x^{3/2} - 5x^{-3} + C$