

3.5 The Derivatives of a^x and $\log_a x$

Recall $\frac{d}{dx} e^x = e^x$

New $\boxed{\frac{d}{dx} a^x = \ln a \cdot a^x}$

Ex $y = 4^x$
 $y' = \ln 4 \cdot 4^x$

Ex $y = 3.2^x$
 $y' = \ln 3.2 \cdot 3.2^x$

Recall $\frac{d}{dx} e^{g(x)} = e^{g(x)} \cdot g'(x)$

New $\boxed{\frac{d}{dx} a^{g(x)} = \ln a \cdot a^{g(x)} \cdot g'(x)}$

Chain Rule

Ex $y = 7^{5x^2-2x}$
 $y' = \ln 7 \cdot 7^{5x^2-2x} \cdot (10x-2)$

Recall $\frac{d}{dx} \ln x = \frac{1}{x}$

(Friday Video #1)

New $\boxed{\frac{d}{dx} \log_a x = \frac{1}{\ln a} \cdot \frac{1}{x}}$

Ex $y = \log_5 x$
 $y' = \frac{1}{\ln 5} \cdot \frac{1}{x}$

Ex $y = 4 \cdot \log x = 4 \cdot \log_{10} x$
 $y' = 4 \cdot \frac{1}{\ln 10} \cdot \frac{1}{x} = \frac{4}{\ln 10} \cdot \frac{1}{x}$

Recall $\frac{d}{dx} \ln(g(x)) = \frac{g'(x)}{g(x)}$ OR $\frac{1}{g(x)} \cdot g'(x)$

Chain Rule

New $\frac{d}{dx} \log_a(g(x)) = \frac{1}{\ln a} \cdot \frac{g'(x)}{g(x)}$ OR $\frac{1}{\ln a} \cdot \frac{1}{g(x)} \cdot g'(x)$

Ex $y = \log_2(x^2 + 3x + 1)$

$$y' = \frac{1}{\ln 2} \cdot \frac{2x+3}{x^2+3x+1} \quad \text{OR} \quad \frac{1}{\ln 2} \cdot \frac{1}{x^2+3x+1} \cdot (2x+3)$$

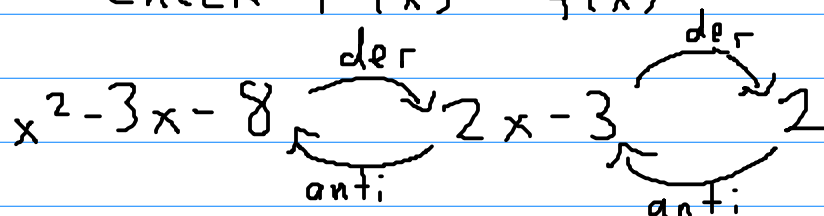
Quiz $y = \log_3(7 - 18x^2)$

$$y' = \frac{1}{\ln 3} \cdot \frac{-36x}{7-18x^2} \quad \text{OR} \quad \frac{1}{\ln 3} \cdot \frac{1}{7-18x^2} (-36x)$$

4.1 Antidifferentiation

Def An antiderivative of a function $f(x)$ is a function $F(x)$ where $F'(x) = f(x)$.

Example an antiderivative of $f(x) = 2x - 3$ is $F(x) = x^2 - 3x - 8$
Check $F'(x) = f(x)$ ✓



But there is an issue.

Another antiderivative of $f(x) = 2x - 3$ is $F(x) = x^2 - 3x + 5$

They differ by a constant.

And $\frac{d}{dx}(\text{constant}) = 0$

The constant can be anything.

Def The general antiderivative of $f(x)$ is $F(x) + C$ where $F'(x) = f$ and C is a generic constant

So the general antiderivative of $f(x) = 2x - 3$ is $F(x) = x^2 - 3x + C$

Notation: The general antiderivative of $f(x)$ is also written

$$\int f(x) dx$$

This is the (indefinite) integral of $f(x)$

$$\int f(x) dx = F(x) + C$$

$$\boxed{\int x^n dx = \frac{1}{n+1} x^{n+1} + C} \quad n \neq -1$$

$$\text{Ex } \int x dx = \frac{1}{2} x^2 + C$$

$$\text{Ex } \int x^3 dx = \frac{1}{4} x^4 + C$$

$$\text{Ex } \int \frac{1}{x^3} dx = \int x^{-3} dx = -\frac{1}{2} x^{-2} + C$$

$$\text{Ex } \int \sqrt{x} dx = \int x^{1/2} dx = \frac{2}{3} x^{3/2} + C$$

$\uparrow$ reciprocal

$$\text{Ex } \int 1 dx = x + C$$

$$\boxed{\int e^x dx = e^x + C}$$

$$\boxed{\int e^{ax} dx = \frac{1}{a} e^{ax} + C}$$

$$\boxed{\int \frac{1}{x} dx = \int x^{-1} dx = \ln x + C}$$

$$\begin{aligned} E_x \int 6x^5 + 6x^2 + 6x + 7 dx \\ = 6 \cdot \frac{1}{6} x^6 + 6 \cdot \frac{1}{3} x^3 + 6 \cdot \frac{1}{2} x^2 + 7x + C \\ = x^6 + 2x^3 + 3x^2 + 7x + C \end{aligned}$$

$$\begin{aligned} E_x \int 15x^{1/2} + 15x^{-4} dx \\ = 15 \cdot \frac{2}{3} x^{3/2} + 15 \left(\frac{-1}{3} \right) x^{-3} + C \\ = 10x^{3/2} - 5x^{-3} + C \end{aligned}$$