3.3/3.4 Exponential Growth and Decay

Interest Compounded Continuously

$$A = P e^{kt}$$

A = end amount
P = Principle / initial deposit
k = appretiation rate
t = time

Ex If \$1000 is invested at a rate of 6% compounded continuously, what is the end amount after 15 years?

$P = 1000 \quad k = 0.06 \quad t = 15$

$$A = 1000 e^{(0.06)(15)}$$

$$= 1000 e^{0.9}$$

$$= 1000 \cdot 2.459$$

$$= \$2,459$$

The Present Value of an investment is the interest formula solved for P.

$$\frac{A}{e^{kt}} = \frac{Pe^{kt}}{e^{kt}}$$

$$P = \frac{A}{e^{kt}}$$

$$\boxed{P = Ae^{-kt}}$$

Ex What is the present value of $1500 payable after 10 years, if money is invested at 4% with interest compounded continuously?

$A = 1500 \qquad t = 10 \qquad k = 0.04$

$$P = 1500 \cdot e^{-(0.04)(10)}$$

$$= 1500\, e^{-0.4}$$

$$= 1500 \cdot 0.67032$$

$$= \$1005.48$$

Doubling Rate: how long does it take for an investment P to double?

take $P = \$100$
$A = \$200$

$$A = Pe^{kt}$$
$$2P = Pe^{kt}$$
$$2 = e^{kt}$$

exp $\longleftrightarrow$ log form

$$kt = \log_e 2$$
$$kt = \ln(2)$$

OR take ln both sides

$$\ln(2) = \ln(e^{kt})$$
$$\ln(2) = kt$$

$$\boxed{T = \frac{\ln(2)}{k}}$$

Ex. At a rate of 3.8%, how long with it take for an account to double?

$$T = \frac{\ln(2)}{0.038} = \frac{0.69314}{0.038} = 18.24 \text{ years}$$

The Doubling Rate can be estimated by the Rule of 70

$$\ln(2) = .693\ldots \sim \frac{70}{100}$$

$$\text{Doubling Time} \approx \frac{70}{100k} = \frac{70}{k \text{ (as a \%)}}$$

Ex (previous example) $\quad \dfrac{70}{3.8} = 18.42$ years

Use log rules to solve for missing variables if necessary.

Ex. What rate is required for an initial investment of $1000 to become $3000 in 10 years?

$A = 3000 \quad P = 1000 \quad t = 10$

$$A = Pe^{kt}$$

$$\frac{3000}{1000} = \frac{1000 \cdot e^{10k}}{1000}$$

$$3 = e^{10k}$$

$$\ln(3) = \ln(e^{10k}) \qquad \ln = \log_e$$

$$\ln(3) = 10k$$

$$\frac{1.986}{10} = \frac{10k}{10}$$

$$k = 0.1986$$
$$= 19.86\%$$