3.2 Loarithmic Functions (part 2)

Properties of Logs (Arithmetic)

- $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$
- $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
- $\log_a(x^n) = n \cdot \log_a(x)$

Ex Given $\log_a 2 = 0.301$, $\log 7 = 0.845$

$$\log_a(14) = \log_a(2 \cdot 7) = \log_a(2) + \log_a(7)$$
$$= 0.301 + 0.845$$
$$= 1.146$$

$$\log_a(16) = \log_a(2^4) = 4 \cdot \log_a 2$$
$$= 4 \cdot 0.301$$
$$= 1.204$$

Properties of Logs (Evaluation)

- $\log_a a = 1 \quad (\Longleftrightarrow a^1 = a)$
- $\log_a 1 = 0 \quad (\Longleftrightarrow a^0 = 1)$
- $\log_a a^n = n \quad (\Longleftrightarrow a^n = a^n)$

$$\log_a\left(\frac{1}{7}\right) = \log_a 1 - \log_a 7$$
$$= 0 - 0.845$$
$$= -0.845$$

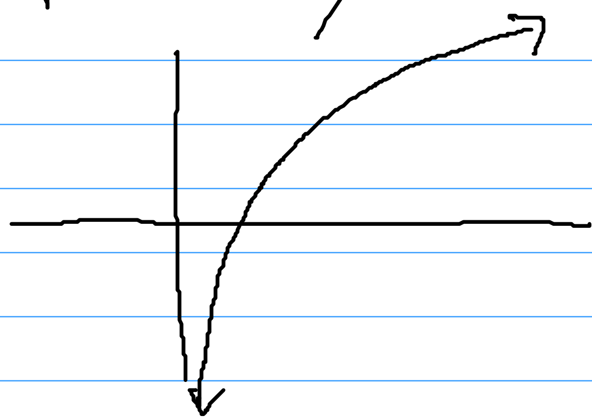
Alternately

$$\log_a\left(\tfrac{1}{7}\right) = \log_a\left(7^{-1}\right) = -1 \cdot \log_a(7)$$
$$= -0.845$$

Def $\log x = \log_{10} x$

Def $\ln x = \log_e x$ Natural Log

Graph of $y = \ln(x)$



Derivative of the Natural Log

If $y = \ln(x)$

then $y' = \dfrac{1}{x}$

Ex $y = x^4 + 3e^x + 2\ln x$

$y' = 4x^3 + 3e^x + \dfrac{2}{x}$

$$\boxed{\frac{d}{dx}\ln(x) = \frac{1}{x}}$$

Ex $\quad y = \underset{u}{x} \cdot \underset{v}{\ln x}$

Use Product Rule

$u = x \qquad\qquad v = \ln x$

$u' = 1 \qquad\qquad v' = \frac{1}{x}$

$$y' = u'v + uv'$$

$$= 1 \cdot \ln x + x\left(\frac{1}{x}\right)$$

$$= \ln x + \frac{\cancel{x}}{\cancel{x}} \qquad = \ln x + 1$$

Chain Rule Shortcut

If $y = \ln\left(g(x)\right)$

then $y' = \frac{1}{g(x)} \cdot g'(x)$ OR $\frac{g'(x)}{g(x)}$

Ex $\quad y = \ln\left(x^3 + 7x\right)$

$$y' = \frac{3x^2 + 7}{x^3 + 7x}$$

Ex $\quad y = \ln(4x)$

$$y' = \frac{4}{4x} = \frac{1}{x}$$

Can use properties of logs to make derivatives easier.

$$y = \ln(4x)$$
$$= \ln(4) + \ln(x)$$

$$y' = 0 + \frac{1}{x} = \frac{1}{x}$$

Ex $y = \ln\sqrt{x-5}$

$$= \ln(x-5)^{1/2}$$

$$= \frac{1}{2}\ln(x-5)$$

$$y' = \frac{1}{2} \cdot \frac{1}{x-5} = \frac{1}{2(x-5)}$$