Written HW #6 Updated

3.1 Exponential Functions

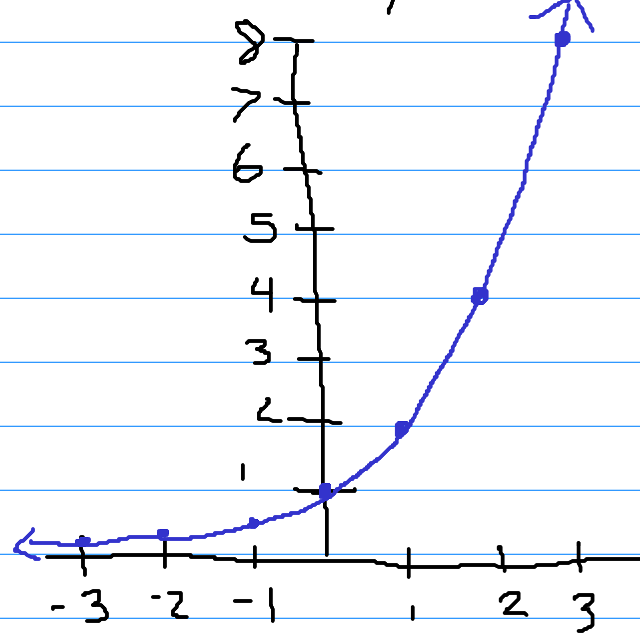
An <u>exponential function</u> has the form

$$f(x) = a^x$$

a is called the <u>base</u> $(a>0,\ a\neq 1)$

Ex graph $f(x) = 2^x$ use xy table

x	y
-3	$2^{-3} = 1/8$
-2	$2^{-2} = 1/4$
-1	$2^{-1} = 1/2$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



e is a naturally arising base which is found in many applications.

$e \approx 2.718281828459045\ldots$

For example: $A = Pe^{kt}$

Derivatives of Exponential Functions

If $y = e^x$
then $y' = e^x$

e^x is its own derivative! $\boxed{\frac{d}{dx}e^x = e^x}$

Ex $y = 3e^x + x^2$

$y' = 3e^x + 2x$

Ex $y = (x^2+3x+1) \cdot e^x$ use Product Rule

$u = x^2+3x+1 \quad v = e^x$

$u' = 2x+3 \quad v' = e^x$

$y' = u'v + uv'$

$= (2x+3)e^x + (x^2+3x+1)e^x$

MML simplifies

$= (2x+3 + x^2+3x+1)\, e^x$

$= (x^2+5x+4)\, e^x$

Ex $y = \dfrac{e^x}{x+2} \begin{matrix} u \\ v \end{matrix}$ use Quot Rule

$u = e^x \quad v = x+2$

$u' = e^x \quad v' = 1$

$y' = \dfrac{u'v - uv'}{v^2}$

$= \dfrac{e^x(x+2) - e^x(1)}{(x+2)^2}$

simplify

$= \dfrac{e^x[x+2-1]}{(x+2)^2}$

$= \dfrac{e^x(x+1)}{(x+2)^2}$

Chain Rule for Exponential Functions

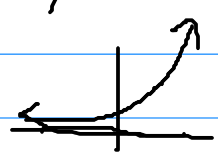
$$\text{If } y = e^{g(x)}$$
$$\text{then } y' = e^{g(x)} \cdot g'(x)$$

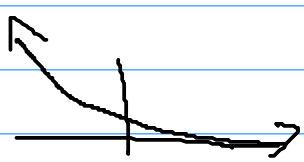
Ex
$$y = e^{x^2+3x-5}$$
$$y' = e^{x^2+3x-5} \cdot (2x+3)$$

Ex
$$y = e^{17x-4}$$
$$y' = e^{17x-4} \cdot 17$$

Ex
$$y = e^{5x^3}$$
$$y' = e^{5x^3} \cdot 15x^2$$

Ex
$$y = e^{8x+12}$$
$$y' = e^{8x+12} \cdot 8$$

e^x  $\lim_{x \to \infty} e^x = \infty$ $\lim_{x \to -\infty} e^x = 0$

e^{-x}  $\lim_{x \to \infty} e^{-x} = 0$ $\lim_{x \to -\infty} e^{-x} = \infty$

3.2 Logarithmic Functions

Exponential Form		Logarithmic Form
$a^n = b$	$\longleftrightarrow$	$n = \log_a b$

$\log_a$ is "log base a"
and a is the base in a^n

$2^3 = 8 \longleftrightarrow 3 = \log_2 8$

$10^4 = 10000 \longleftrightarrow 4 = \log_{10} 10000$

$2^{-1} = \frac{1}{2} \longleftrightarrow -1 = \log_2 \left(\frac{1}{2}\right)$

Properties of Logs (Arithmetic)

- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
- $\log_a(x^n) = n \log_a x$

Given that $\log_a 2 = 0.301$
$\log_a 7 = 0.845$

Find $\log_a 14$

$= \log_a(2 \cdot 7) = \log_a 2 + \log_a 7 = 0.301 + 0.845$
$= 1.146$

Find $\log_a 16$

$= \log_a(2^4) = 4 \cdot \log_a(2) = 4(.301)$
$= 1.204$