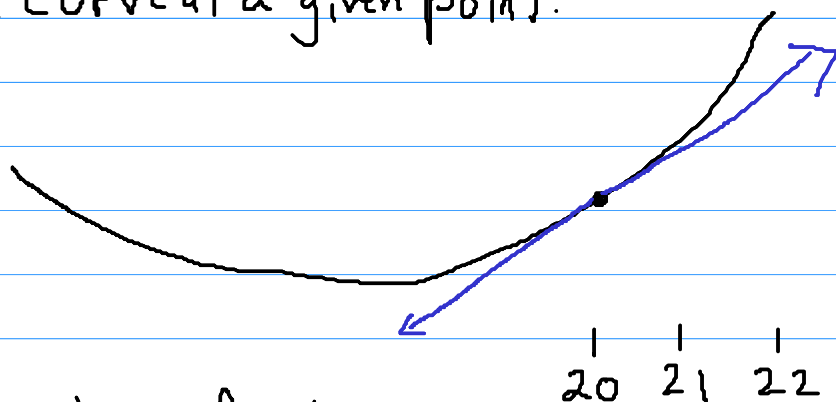
Test 2
Monday Oct 26
RH 1558
6-7pm

2.6 Marginals and Differentials

Recall that the tangent line is the line that most closely a curve at a given point.



The y value of the tangent line is close to the y value of f(x) at $x=21$.

Idea: $f'(20) \approx \dfrac{f(21)-f(20)}{21-20}$

$f'(20) \approx f(21) - f(20)$

OR $f(21) \approx f(20) + f'(20)$

For $R(x)$ = revenue, $R(N)$ = revenue from selling N

$R(N+1)$ = " " " $N+1$

$R(N+1) - R(N)$ = additional revenue for selling one more item past N.

Def <u>Marginal Revenue</u> is the approximation

$$R'(N) \approx R(N+1) - R(N)$$

Similarly $C'(N)$ = Marginal Cost at N^{th} item

$P'(N)$ = Marginal Profit at N^{th} item

Use Marginal Cost/Revenue/Profit to estimate future C/R/P.

$$R(N+1) \approx R(N) + R'(N)$$

Ex If $R(150) = \$700$
$R'(150) = \$10$

Estimate $R(151)$
$\approx R(150) + R'(150)$
$700 + 10$
$\$710$

Estimate $R(152)$ $R(150+2)$
$\approx R(150) + 2 \cdot R'(150)$
$700 + 2 \cdot 10$
$\$720$

Ex $P(x) = -7x^2 + 600x + 40$

Find $P'(x)$
$P'(x) = -14x + 600$

Find $P(50)$
$\$12540$

Find $P'(50)$
-100

Estimate $P(51)$
$\$12{,}540 - 100 = \$12{,}440$

2.7 Elasticity of Demand

How does a change in price affect a change in demand?

x = price per unit
$q = D(x)$ = demand at price x

Derivation:

If Δx = change in price
then $\Delta x / x$ = percent change in price

Ex Changing from \$20 to \$22
Δx = change $= 22 - 20 = 2$
$\Delta x / x = 2/20 = .10 = 10\%$ increase

Also Δq = change in quantity
$\Delta q / q$ = percent change in quantity

Ratio of percents of change

$$= \frac{\Delta q / q}{\Delta x / x} = \frac{x}{q} \cdot \frac{\Delta q}{\Delta x}$$

Limit as $\Delta x \to 0$

$$= \frac{x}{q} \cdot \frac{dq}{dx}$$

$$= \frac{x}{D(x)} \cdot D'(x)$$

Elastricity of Demand $E(x)$

$$\boxed{E(x) = -\frac{x \cdot D'(x)}{D(x)}}$$

① $D'(x)$ is negative because demand decreases as price increases.

② The negative sign makes $E(x)$ positive since $D'(x)$ is negative.

Ex $q = D(x) = 700 - 2x$

a) Find demand when price = \$30

$$D(30) = 700 - 2(30) = 640$$

b) Find elasticity of demand.

Need $D'(x) = -2$

$$E(x) = \frac{-x \cdot D'(x)}{D(x)}$$

$$= \frac{-x(-2)}{700-2x}$$

$$= \frac{2x}{700-2x}$$

c) Find elasticity of demand at \$50

$$E(50) = \frac{2(50)}{700-2(50)} = \frac{100}{600} = \frac{1}{6}$$

at \$25

$$E(25) = \frac{2(25)}{700-2(25)} = \frac{50}{650} = \frac{1}{13}$$

$R(x)$ = Revenue = (price)(demand) = $x \cdot D(x)$

$$R'(x) = D(x) \cdot [1 - E(x)]$$ (see book for proof)

Theorem

Revenue is increasing (demand is inelastic) if $E(x) < 1$

Revenue is decreasing (demand is elastic) if $E(x) > 1$

Revenue is maximized (unit elasticity) if $E(x) = 1$

Ex Find unit elasticity in previous problem.

$$E(x) = 1$$

$$\frac{2x}{700-2x} = 1$$

$$2x = 700 - 2x$$

$$4x = 700$$

$$x = \$175$$