2.5 Business and Economics Applications

Max/Min of Cost/Revenue/Profit

Ex. The cost of selling x clocks is (in dollars)

$$C(x) = 10x + 3$$

The revenue from selling x clocks is

$$R(x) = 50x - 0.5x^2$$

a) Find the equation for profit

$$P(x) = R(x) - C(x)$$
$$= 50x - 0.5x^2 - [10x + 3]$$
$$= -0.5x^2 + 40x - 3$$

b) Find the number of clocks to sell to maximize profit

$$P'(x) = -x + 40$$
$$-x + 40 = 0 \qquad \text{(critical point)}$$
$$x = 40$$

c) Find the maximum profit

$$P(40) = -800 + 1600 - 3$$
$$= \$796$$

d) Verify that it is a maximum

Use 1st or 2nd derivative test

$$P''(x) = -1$$
$$P''(40) = -1 \quad \frown \quad \text{maximum} \checkmark$$

Optimizing Item Price

As the price of an item increases,
the number sold at that price decreases.

Goal: Find the price/item which maximizes profit

Ex To sell x tubas, the price per tuba must be

$$p = 1000 - x$$

The cost for producing x tubas is

$$C(x) = 2000 + 300x$$

a) Find the revenue

$$R(x) = p \cdot x = (1000 - x) \cdot x$$
$$= 1000x - x^2$$

b) Find the profit

$$P(x) = R(x) - C(x) = 1000x - x^2 - [2000 + 300x]$$
$$= -x^2 + 700x - 2000$$

c) When is profit maximized?

$$P'(x) = -2x + 700$$
$$-2x + 700 = 0 \qquad (P'(x) = 0)$$
$$x = 350$$

d) Find max profit

$$P = -122{,}500 + 245{,}000 - 2{,}000$$
$$= \$120{,}500$$

e) Find optimal price

$$p = 1000 - 350$$
$$= \$650/\text{tuba}$$

Ex Tickets for a Football Game

When tickets are \$21 each, 8000 people attend.
For every decrease of \$2, 1000 more people attend.

Also, each person spends an average of \$5 on concessions.

Find the ticket price that maximizes revenue.

$x = \#$ of decreases

a) Find the price/ticket with x decreases.

$p = 21 - 2x$

b) Find the number of total people with x decreases.

$8000 + 1000x$

c) Find the revenue R(x) = ticket sales + concession sales

$$= (21-2x)\cdot(8000+1000x) + 5\cdot(8000+1000x)$$
$$= 168{,}000 - 16{,}000x + 21{,}000x - 2{,}000x^2 + 40{,}000 + 5{,}000x$$
$$= -2{,}000x^2 + 10{,}000x + 208{,}000$$

d) For what x is R(x) maximized?

$$R'(x) = -4{,}000x + 10{,}000 = 0 \qquad \text{(critical point)}$$
$$x = \frac{10{,}000}{4{,}000} = 2.5$$

e) Find the optimal price/ticket and expected # of people

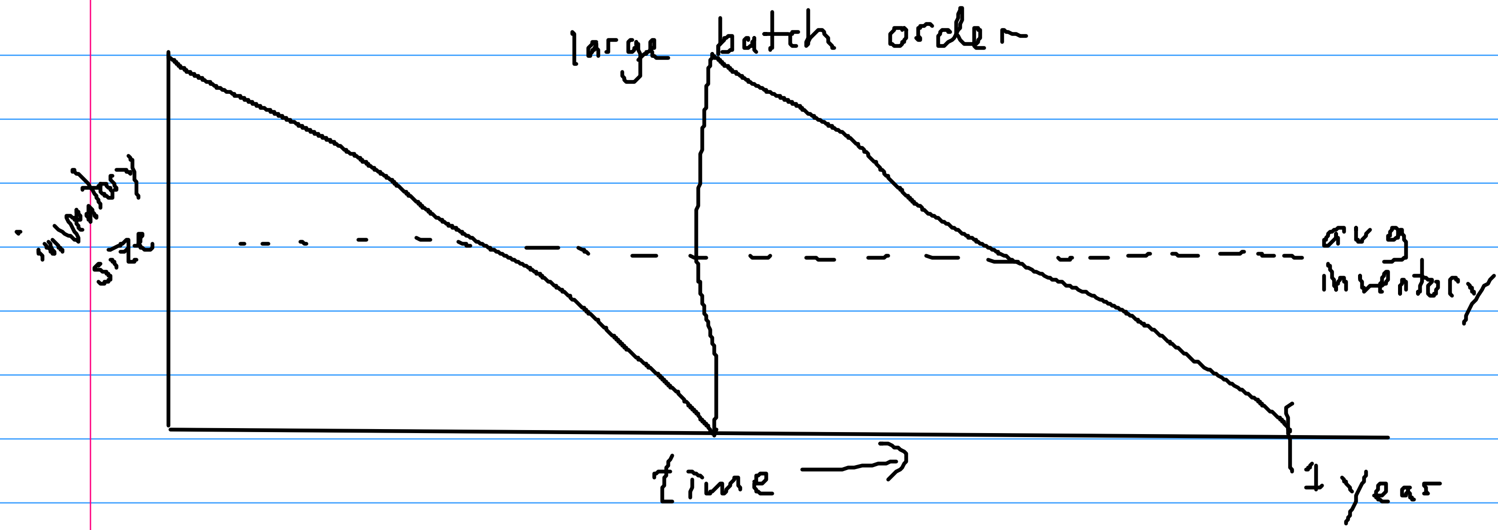
$p = 21 - 2(2.5)$
$= \$16$

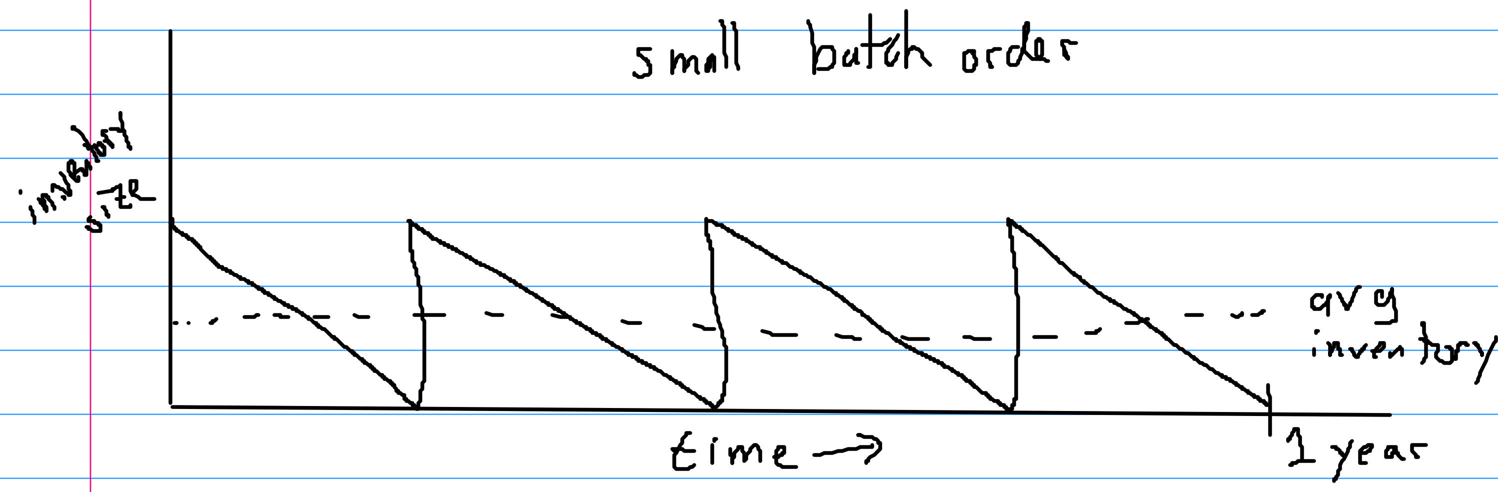
$8000 + 1000(2.5)$
$= 10{,}500$ people

Minimizing Inventory Costs

There is a cost associated with maintaining an inventory of items to sell.

If the yearly perchase rate is fixed, should one order
- few large batches? OR
- many small batches?





Let x = number of units in an order

$\frac{x}{2}$ = average number of units in inventory

Equation to minimize:

(Total Cost) = (Storage Cost) + (Reorder Cost)

Ex. A store sells 180 chairs per year.
It costs $10 to store one chair for a year.
To reorder, there is a fixed cost of $100, plus $50 per chair.

Find the optimal order size, and number of orders per year.

(Total Cost) = (Storage Cost) + (Reorder Cost)

For x chairs in an order

$$C(x) = (\text{storage cost/chair}) * (\text{avg \# of chairs}) + (\text{cost per order}) * (\text{number of orders})$$

$$= (10)\left(\frac{x}{2}\right) + (100 + 50x)\left(\frac{180}{x}\right)$$

$$= 5x + \frac{18000}{x} + \frac{9000\cancel{x}}{\cancel{x}}$$

$$= 5x + 18000x^{-1} + 9000$$

minimize $C(x)$

$$C'(x) = 5 - 18000x^{-2}$$

$$5 - \frac{18000}{x^2} = 0$$

$$5 = \frac{18000}{x^2}$$

$$5x^2 = 18000$$

$$x^2 = 3600$$

$x = \sqrt{x^2} = \sqrt{3600} = 60$ order size

number of orders in a year $= \frac{180}{60} = 3$