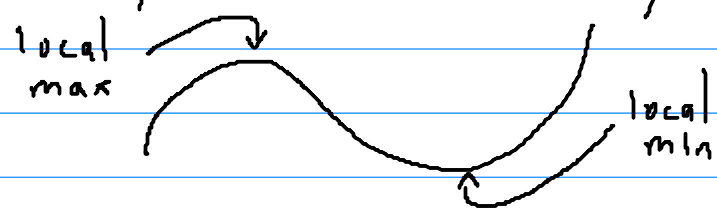
Test 2
Oct 26

2.4 Using Derivatives to Find Absolute Maximum and Minimum Values

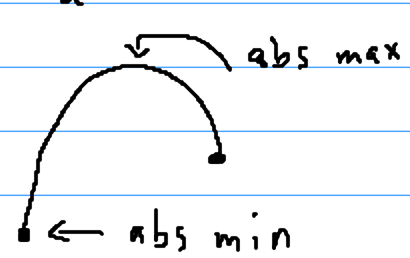
Recall: Relative/local extrema $x=c$ is a local max (or min) if $f(c)$ is larger (or smaller) than any $f(x)$ for nearby x.

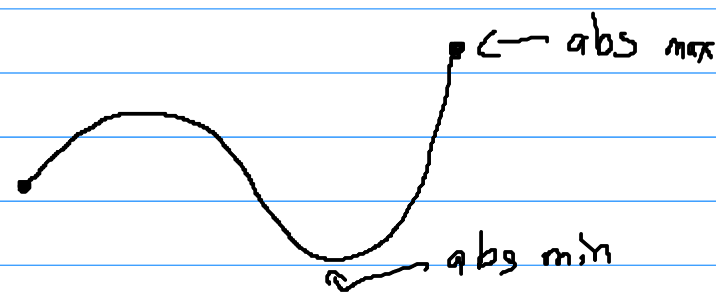


Absolute Extrema on Closed Interval $[a,b]$:
$x=c$ is the absolute max (or min) if $f(c)$ is larger (or smaller) than <u>every</u> $f(x)$ in the interval $[a,b]$

Note: A function can have at most one abs. max or min.

Ex





Notice the absolute extrema occur at
- critical values (relative extrema)

or - endpoints

To find abs. max and min:

Find
- critical pts ($f' = 0$)
- endpts

Throw out x values that are outside the interval.

Evaluate x values on $y = f(x)$ (for heights)

Find the abs max and min

Ex $f(x) = x^2 - 6x + 2$ on $[0, 4]$

find abs max and min

critical pts ($f' = 0$):

$$f' = 2x - 6 = 0$$
$$x = 3$$

end pts: $x = 0, 4$

← all are in the interval

heights ($y = f(x)$):

$f(0) = 2$ ← abs max

$f(3) = -7$ ← abs min

$f(4) = -6$

Abs max of 2 at $x = 0$

Abs min of -7 at $x = 3$

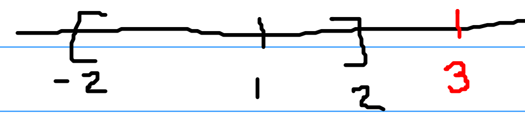
Ex $f(x) = x^3 - 6x^2 + 9x - 1$ on $[-2, 2]$, find the abs max and min

critical pts $(f' = 0)$:

$$f' = 3x^2 - 12x + 9 = 0$$
$$3(x^2 - 4x + 3) = 0$$
$$3(x-1)(x-3) = 0$$

$x = 1$, $\cancel{3}$ not in interval

end pts: $x = -2, 2$



Heights:

$f(-2) = -51$ ← abs min
$f(1) = 3$ ← abs max
$f(2) = 1$

Abs max of 3 at $x = 1$
Abs min of -51 at $x = -2$

2.5 Maximum-Minimum Problems; Business and Economic Applications

Max/Min of Cost/Revenue/Profit:

The cost of selling x clocks is

$$C(x) = 10x + 3$$

The revenue from selling x clocks is

$$R(x) = 50x - 0.5x^2$$

Find the equation for profit

$$P(x) = R(x) - C(x)$$
$$= 50x - 0.5x^2 - (10x + 3)$$
$$= 50x - 0.5x^2 - 10x - 3$$
$$= 40x - 0.5x^2 - 3$$

Find the number of clocks to sell to maximize the profit.

$P' = 0$ critical pts

$$P' = 40 - x = 0$$
$$x = 40 \text{ clocks}$$

Find the maximum profit.

$$P(40) = \$797$$

Verify that it is a maximum (2nd der test)

$P'' = -1$, $P''(40) = -1 < 0$ ∩ max

Optimize Item Price

As the price of an item increases, the number sold at that price decreases.

Goal: find the price/item to use to maximize profit.

Ex To sell x tubas, the prize needs to be

$$p = 1000 - x$$

The cost for producing x tubas is

$$C(x) = 2000 + 300x$$

Find revenue $R(x)$

$$R(x) = \text{quantity} \times \text{price}$$
$$= x(1000 - x)$$
$$= 1000x - x^2$$

Find Profit $P(x)$

$$P(x) = R(x) - C(x)$$
$$= -x^2 + 700x - 2000$$

When is Profit maximized?

$$P' = 0$$
$$-2x + 700 = 0$$

$x = 350$ tubas

Find optimal price

$$p = 1000 - 350$$
$$= \$650$$