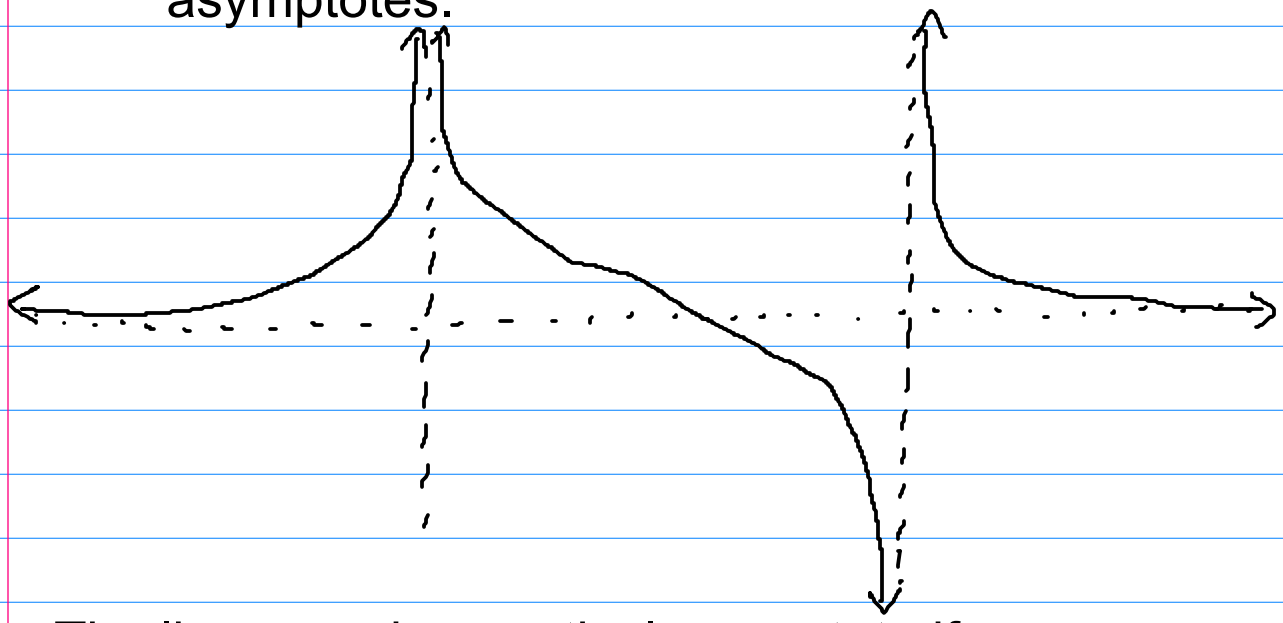
2.3 Graph Sketching:
Asymptotes and Rational Functions

A rational function is a function of the form $f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials.

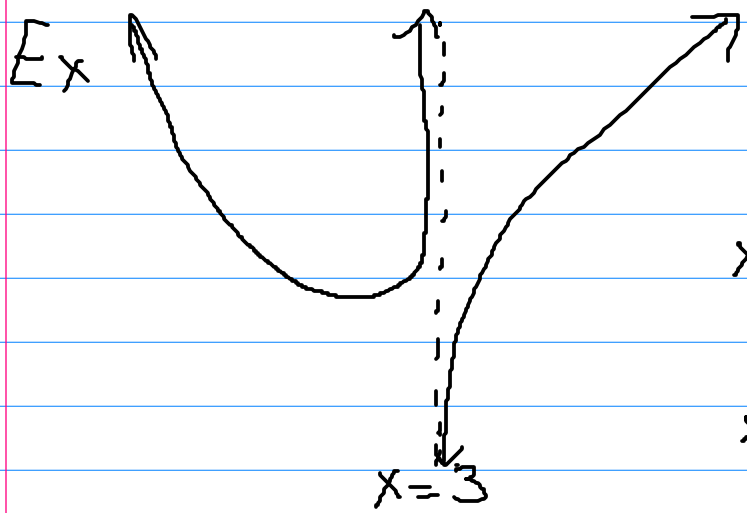
Rational functions can have vertical and horizontal asymptotes.



The line $x = a$ is a vertical asymptote if

$$\lim_{x \to a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^+} f(x) = \pm\infty$$

Ex



$$\lim_{x \to 3^-} f(x) = +\infty$$

$$\lim_{x \to 3^+} f(x) = -\infty$$

Vertical asymptotes for a rational function occur in the simplified (or reduced) version where the denominator = 0.

Ex Find the vertical asymptotes

$$f(x) = \frac{x+3}{x-5}$$

$x - 5 = 0$

$x = 5$ V.A.

$$f(x) = \frac{x^2 - 5x + 6}{x^2 - 4} = \frac{(x-2)(x+3)}{(x-2)(x+2)}$$

$x + 2 = 0$

$x = -2$ V.A.

other soln $x = 2$ is a hole in the graph

The line y = b is a horizontal asymptote if

$$\lim_{x \to \infty} f(x) = b \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b$$

Ex

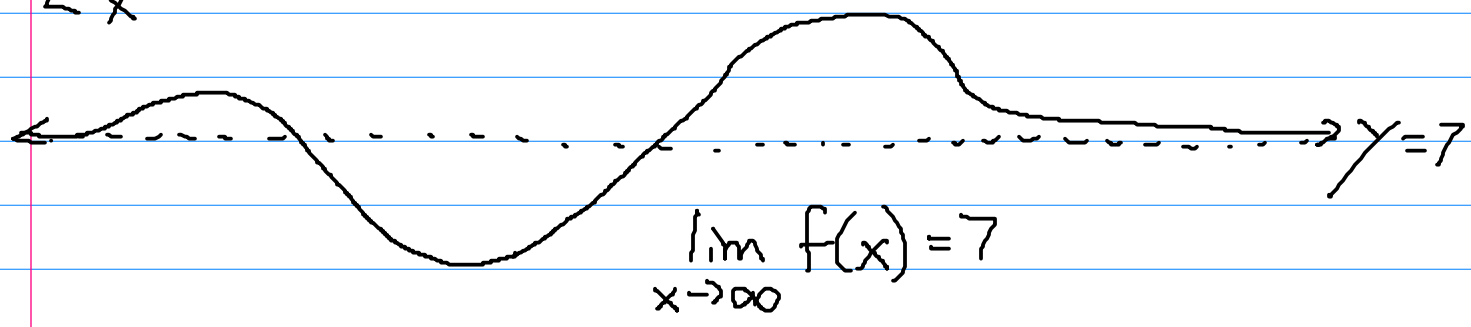


Table Method for finding H.A.

$x \to \infty$

x	$f(x)$
10	·
100	·
1000	·
10000	·

goes to ∞

$x \to -\infty$

x	$f(x)$
-10	·
-100	·
-1000	·
-10000	·

Ex $\lim_{x \to \infty} \frac{1}{x}$

x	$f(x)$
10	$1/10 = .1$
100	$1/100 = .01$
1000	$1/1000 = .001$
10000	$1/10000 = .0001$

∞ ... 0

New Limit Laws:

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} \frac{1}{x^n} = 0$$

Algebraic Method for finding H.A.

Let x^n be the highest power of x in the denominator
Multiply numerator and denominator both by $\frac{1}{x^n}$
and distribute.
Use the Limit Laws to find the limit.

Ex $\lim_{x\to\infty} \frac{2x^2-1}{3x^2+7x}$

$= \lim_{x\to\infty} \frac{2x^2-1}{3x^2+7x} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{\frac{2x^2}{x^2} - \frac{1}{x^2}}{\frac{3x^2}{x^2} + \frac{7x}{x^2}}$

$= \lim_{x\to\infty} \frac{2 - \frac{1}{x^2}}{3 + \frac{7}{x}}$

$= \frac{2-0}{3+0}$

$= \frac{2}{3}$

Must use this method to get full credit.
Other methods can be used to check your answer.

Ex $\lim_{x\to\infty} \frac{4x+3}{x^2+2x+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{\frac{4}{x} + \frac{3}{x^2}}{1 + \frac{2}{x} + \frac{1}{x^2}}$

$= \frac{0+0}{1+0+0}$

$= 0$

Ex $\lim_{x\to\infty} \frac{x^2}{x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{x}{1+\frac{1}{x}}$

$= \frac{\infty}{1+0}$

$= \infty$

Recall: Horizontal Asymptotes of f(x) = $\lim_{x\to\infty} f(x)$

There are no slant limits in the homework.